DIFFERENTIAL EQUATIONS
AND
CONTROL PROCESSES
N. 3, 2020
Electronic Journal,
reg. N ФС77-39410 at 15.04.2010
ISSN 1817-2172

http://diffjournal.spbu.ru/
e-mail: jodiff@mail.ru

Adaptive and robust control

Full control of a quadrotor with Simple Adaptive Control

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Abstract. In the paper the Simple Adaptive Control approach is employed to design the adaptive controllers for controlling both angular and translational motion. The adaptive controllers are synthesized based on the Implicit Reference Model (IRM) design technique. The "shunting method" (parallel feedforward compensation) is used to mitigate the relative degree restriction. Parametric feedback is used to prevent the increasing of controller coefficients caused by external disturbances or sensor noise. Quality of the closed-loop IRM adaptive control system is studied based on the series of simulations for ensuring the system stability and tracking cases.

Keywords: quadrotor, simple adaptive control, passification, cyber-physical system.

1 Introduction

In the last decade, interest in using UAVs (unmanned aerial vehicles) of the helicopter type (hexacopters, quadrotors, etc.) has increased significantly in various fields, such as surface, environmental and traffic monitoring, studying of atmospheric flows and etc. In many missions, the parameters of the UAV are subject to significant changes or a priori unknown. This happens, for example, due to a change in load from one flight to another flight or changes may occur during a UAV flight depending on external environment properties, i.e. windage. The presence of cranes or mobile manipulators, which are mounted on the UAV, also leads to a significant change in the UAV parameters. As a result, in the recent years a significant amount of work has been done on the use of adaptation methods for designing multirotors mini-UAV control systems. Many factors can affect to changing the UAV dynamics and, as a consequence, the control quality. Some applications require adding some physical load such as video cameras, packages and etc to quadrotors. So it is very important to use control system that could be flexible and adaptive for plants parameter variations.

Two types of nonlinear controllers for an autonomous quadrotor helicopter are presented in [22]: feedback linearization controller and an adaptive sliding mode controller using input augmentation to take into account the helicopter underactuated property. It has been demonstrated that the sliding mode controller is efficient in the case of ground effects.

The approaches above mainly lead to quite complex adaptive control laws. In the present work, the *Simple Adaptive Control* (SAC) method, jointly with "*shunting*" (*parallel feedforward compensation*) is used for controlling both angular and translational motion. We deal with the ultralight quadrotor, which is a part of the research complex *QuadRoy* (*Swarm of Quadrotors*), see [1, 4, 29, 31, 32], which was developed in the IPME RAS. The quadrotors are supplied with autopilot Pixhawk, the Inertial Navigation System (including gyroscope, accelerometer, magnetometer, barometer) and the GPS receiver.

The SAC approach is being intensively developed since the 1970s, see [2, 5, 8–10, 26, 30], for mentioning a few. Among the advantages of this method as opposed to the conventional *Model Reference Adaptive Control* (MRAC, [20]) method are low order and computational simplicity of the control algorithms and the weakening the restrictions imposed by the so-called *matching* (*Erzberger*) *condition*, see [18]. The SAC method has been successfully applied to the various adaptive control problems, such as control of power systems [36], robotic systems [25,27], spacecrafts [6,21,33], aircraft [15,34], fault tolerant control [14,19], etc. SAC method was designed and successfully used for quadrotor attitude control [7,31]. Computational simplicity is essential for small quadrotor autopilots because most autopilots are equipped with low powerful microprocessors for computing control signals, processing sensors data, sending telemetry over communication channels, etc.

The rest of the paper is organized as follows. Firstly, the SAC scheme based on the passification method is briefly recalled in Sec. 2. The SAC laws for control both angular and translational motion are derived in Sec. 3. The simulation results of testing proposed algorithms are presented in Sec. 4. Concluding remarks and the future work intentions finalize the paper.

2 Adaptive Controllers With Implicit Reference Model

A simple tool for SAC design is the *Passification method*. Passification means rendering the closed loop system passive by output feedback, see [2, 10, 13] for the details. The main condition following from this method lies in the so-called *hyper minimum phase* (HMP) property, imposed on the plant model: $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t)$, where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^m$, A, B, C are constant real matrices of appropriate dimensions. According to this property polynomial $\varphi_0(s) = \det \begin{bmatrix} sI_{\mathrm{n}} - A & -B \\ C & 0 \end{bmatrix}$ where I_{n} stands for an $(n \times n)$ identity matrix, must be Hurwitz and $CB = (CB)^{\mathrm{T}} > 0$. For the special case $m = 1$ (SISO systems) considered in this paper the HMP property coincides with the standard minimum-phase, relative-degree-one property. Passification method makes it possible to create SAC with the *Implicit Reference Model* (IRM), of an order that can be significantly less than that of the plant model.

2.1 Adaptive Stabilization of LTI SISO Plants

Let LTI SISO plant be modeled in the input-output form as

$$A(p)y(t) = B(p)u(t), \quad t \geq 0, \tag{1}$$

where u, y are scalar input and output variables, $A(p) = p^n + a_{n-1}p^{n-1} + \cdots + a_0$, $B(p) = b_m p^m + \cdots + b_0$ are polynomials on time derivative operator $p \equiv d/dt$. Define k as the *relative degree* of system (1), $k = n - m > 0$. Plant (1) parameters a_i, b_j ($i = 0, \ldots, n-1$, $j = 1, \ldots, m$) are assumed to be unknown. Desired closed-loop system performance may be expressed in the form of a certain “reference” differential equation. In the classical MRAC this equation is explicitly implemented in the adaptive controller by the Reference Model, cf. [20]. To describe the IRM adaptive controllers, let us introduce an *adaptation error* signal $\sigma(t)$ as $\sigma(t) = G(p)y(t)$, where $G(p) = p^l + g_{l-1}p^{l-1} + \cdots + g_0$ is a given Hurwitz polynomial in operator $p \equiv d/dt$. Coefficients g_i are the design parameters; they are chosen based on the desired dynamics of the closed-loop system. Degree l of polynomial $G(p)$ is defined below. Assuming that the adaptation law ensures tendency $\sigma(t)$ to zero let us notice that as $\sigma \equiv 0$ output $y(t)$ satisfies the following "*reference equation*"

$$G(p)y(t) = 0, \tag{2}$$

describing the reference model. This model is not implemented in the adaptive controller in the form of a certain dynamical subsystem, but introduced *implicitly* via parameters g_i $(i = 0, 1, \ldots, l-1)$ [35]. Therefore it is called the Implicit Reference Model (IRM).

Let us choose the feedback control law in the following form: $u(t) = \sum\limits_{i=0}^{l} k_i(t)\big(p^i y(t)\big)$, where $k_i(t)$ $i = 0, \ldots, l$ are adjustable controller parameters. For the considered case the IRM design method leads to the following adaptation law, see [10, 11]:

$$\dot{k}_i(t) = -\gamma\sigma(t)p^i y(t), \quad k_i(0) = k_i^0, \tag{3}$$

where $\gamma > 0$ is the *adaptation gain*, k_i^0 are given initial values of the controller gains, $i = 0, \ldots, l$. Introducing row vector $G = [g_0, g_1, \ldots, 1] \in \mathbb{R}^{l+1}$ and plant (1) transfer function $W(s)$ from input u to output vector $[y, \dot{y}, \ldots y^{(l)}]^{\mathrm{T}} \in \mathbb{R}^{l+1}$ as $W(s) = \dfrac{B(s)}{A(s)}\left[1, s, \ldots, s^l\right]^{\mathrm{T}}$, $s \in \mathbb{C}$ in virtue of passification theorem [2, 10] with respect to transfer function $GW(s)$ one may easily obtain the following stability conditions of adaptive controller (3):

1. polynomial $B(s)$ is Hurwitz and $b_0 > 0$;
2. $l = k - 1$, where $k = n - m$ is a relative degree of plant model (1).

Algorithm (3) usually ensures vanishing $\sigma(t)$ essentially faster than transients in the closed-loop. As a result, changing controller (3) gains is stopped and plant (1) output $y(t)$ obeys the IRM (2).

To avoid unlimited growth of controller (3) gains in the presence of external disturbances and measuring errors, the following α-modification of (3) may be used, see [2, 17]:

$$\dot{k}_i(t) = -\gamma\sigma(t)p^i y(t) - \alpha\big(k_i(t) - k_i^0\big), \quad k_i^0 = k_i(0), \tag{4}$$

where *parametric feedback gain* $\alpha \geq 0$ is introduced.

2.2 Adaptive Tracking Systems with IRM

Adaptive control law (4) may be straightforwardly extended to solving the tracking problem with the desired closed-loop system dynamics, see [2]. To this end let us introduce reference signal $r(t)$ and define adaptation error signal $\sigma(t)$ as

$$\sigma(t) = G(p)y(t) - D(p)r(t), \tag{5}$$

where $G(p)$ is defined above, and operator polynomial $D(p)$ has the form $D(p) = d_q p^q + \cdots + d_1 p + d_0$. Signal $\sigma(t)$ may be treated as the discrepancy in the equation

$$G(p)y(t) = D(p)r(t), \tag{6}$$

considering (6) as the IRM for the case of tracking system.

By the analogy with (3) let us take the control action in the form

$$u(t) = k_r(t)\big(D(p)r(t)\big) + \sum_{i=0}^{l} k_i(t)\big(p^i y(t)\big), \tag{7}$$

where $k_r(t)$, $k_i(t)$ $(i = 0, \ldots, l)$ are tunable parameters, and take the following adaptation law

$$\begin{aligned} \dot{k}_r(t) &= \gamma\sigma(t)D(p)r(t) - \alpha\big(k_r(t) - k_r^0\big), \quad k_r^0 = k_r(0), \\ \dot{k}_i(t) &= -\gamma\sigma(t)p^i y(t) - \alpha\big(k_i(t) - k_i^0\big), \quad k_i^0 = k_i(0), \end{aligned} \tag{8}$$

where $\gamma > 0$, $\alpha \geq 0$ are design parameters; k_r^0, k_i^0 are "guessed" initial controller gains, $i = 0, \ldots, l$. It worth mentioning that both degree q of polynomial $D(p)$ and its coefficients may be chosen arbitrarily by the designer.

2.3 Signal-Parametric Adaptive Controllers with IRM

Let the regulation goal $\lim_{t\to\infty} x(t) = 0$ for plant

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t), \tag{9}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^m$, A, B, C are constant real matrices of appropriate dimensions, be posed. Following [2, 16] introduce an auxiliary goal as ensuring the *sliding mode* motion along the predefined surface (line in the scalar control case), which is given by the equality $\sigma(t) \equiv 0$, where $\sigma(t) = Gy(y)$, G is a given $(l \times n)$-matrix. Let us use the following control law:

$$u = -\gamma \operatorname{sign} \sigma, \quad \sigma = Gy, \tag{10}$$

where $\gamma > 0$ is a controller parameter. It may be proved that for system (9), (10) the posed control goal may be achieved if there exist matrix $P = P^{\mathrm{T}} > 0$ and vector K_* s.t. $PA_* + A_*{}^{\mathrm{T}}P < 0$, $PB = GC$, $A_* = A + BK_*{}^{\mathrm{T}}C$. As follows from the passification theorem, the mentioned conditions are fulfilled iff: transfer function $GW(s)$ is HMP (where $W(s) = C\big(\lambda I_n - A)^{-1}B)$; the sign of GCB is known (we assume that it is positive). Under these conditions the goal $\lim\limits_{t\to\infty} x(t) = 0$ is achieved for sufficiently large γ (with respect to the initial conditions and actual plant parameters).

To avoid dependence of closed-loop system stability on initial conditions and plant parameters, the following "*signal-parametric*" ("*combined*") control law may be used instead of (10):

$$\begin{aligned} &u = K^{\mathrm{T}}(t)y(t) - \gamma\,\mathrm{sign}\,\sigma(y), \quad \sigma(y) = G\,y(t) \\ &\dot{K}(t) = -\sigma(y)\Gamma y(t), \end{aligned} \tag{11}$$

where $\Gamma = \Gamma^{\mathrm{T}} > 0$, $\gamma > 0$ are the matrix and scalar adaptation gains (design parameters).

For the case of scalar control input the following control law, inspired by [23, 28], may be used instead of (11):

$$\begin{aligned} &u = -k(t)\sigma(y) - \gamma_\sigma \mathrm{sign}(\sigma(y))\sqrt{|\sigma(y)|}, \quad \sigma(y) = Gy, \\ &\dot{k}(t) = \gamma_k \sigma(t)^2. \end{aligned} \tag{12}$$

This law produces more smooth control action than the "relay" one.

3 Simple Adaptive Control Of Quadrotor

Let us apply the IRM technique for designing the adaptive attitude control algorithms for quadrotor. This technique will be used for translational motion.

3.1 Quadrotor Model

In the present paper the following model of the quadrotor rotational motion dynamics is utilized, cf. [30]:

$$\begin{cases} m\dot{V}_x = \tau_y(C_\psi S_\vartheta C_\gamma + S_\psi S_\gamma) - V_x A_x, \\ m\dot{V}_y = -mg + \tau_y(C_\vartheta C_\gamma) - V_y A_y, \\ m\dot{V}_z = \tau_y(S_\psi S_\vartheta C_\gamma - C_\psi S_\gamma) - V_z A_z, \\ \dot{\gamma} = \omega_x + S_\gamma T_\vartheta \omega_z + C_\gamma T_\vartheta \omega_y, \\ \dot{\vartheta} = C_\gamma \omega_z - S_\gamma \omega_y, \\ \dot{\psi} = \frac{S_\gamma}{C_\vartheta}\omega_z + \frac{C_\gamma}{C_\vartheta}\omega_y, \\ I_x\dot{\omega}_x = (I_y - I_z)\omega_y\omega_z - I_r\omega_z\omega_r + \tau_\gamma, \\ I_y\dot{\omega}_y = (I_z - I_x)\omega_x\omega_z + \tau_\psi, \\ I_z\dot{\omega}_z = (I_x - I_y)\omega_x\omega_y + I_r\omega_x\omega_r + \tau_\vartheta, \end{cases} \tag{13}$$

where γ, ϑ, ψ denote the Euler angles (roll, pitch and yaw, respectively); ω_x, ω_y, ω_z are angular rates in body-axis frame, $S_\xi = \sin(\xi)$, $C_\xi = \cos(\xi)$, $T_\xi = \tan(\xi)$; g is the gravity acceleration; I_x, I_y, I_z are rotational moments of inertia. Rotating torque with respect to CoG is a vector $\tau = \left[\tau_\gamma, \tau_\vartheta, \tau_\psi\right]^{\mathrm{T}}$ with components $\tau_\gamma = l(\omega_2 - \omega_4), \tau_\vartheta = l(\omega_1 - \omega_3)$, $\tau_\psi = a_r\omega_R$, where $\omega_R = \omega_1 - \omega_2 + \omega_3 - \omega_4$. Motor rotation velocity is as $\dot{\omega}_i = k_l(\tilde{\omega}_i - \omega_i)$, where k_l is motor parameter, $\tilde{\omega}_i$ input reference value. Rotation velocities are limited by inequalities: $0 < \omega_{\min} \leq \omega_i \leq \omega_{\max}$. Since k_l is large, the engine inertia is omitted in the sequel.

3.2 Adaptive Control Laws for Quadrotor Attitude

Assuming that angular velocities ω_x, ω_y, ω_z are small, one may obtain the linearized model of quadrotor angular motion in the vicinity of zero. In this model the yaw motion may be considered separately of the roll and pitch motions and modeled by the following equation: $\ddot{\psi} = I_y^{-1}\tau_\psi$ or in the state-space form with state vector $\tilde{x}_\psi = \left[\psi, \dot{\psi}\right]^{\mathrm{T}}$ as

$$\dot{\tilde{x}}_\psi = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tilde{x}_\psi + \begin{bmatrix} 0 \\ I_y^{-1} \end{bmatrix} \tau_\psi, \qquad \tilde{y}_\psi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tilde{x}_\psi. \tag{14}$$

Following the aforementioned IRM design technique, let us introduce vector G_ψ as $G_\psi = \left[1, \alpha_\psi\right]$ where $\alpha_\psi > 0$ is a design parameter. This leads to the following algorithm for yaw control:

$$
\begin{aligned}
&\sigma_\psi(t) = \psi(t) - \psi^*(t) + \alpha_\psi \dot{\psi}(t),\\
&u_\psi(t) = -k_\psi(t)\sigma_\psi(t) - \gamma_\sigma \operatorname{sign}(\sigma_\psi(t))\sqrt{|\sigma_\psi(t)|},\\
&\dot{k}_\psi(t) = \gamma_k \sigma_\psi(t)^2 - \alpha_k\big(k_\psi(t) - k_\psi^0\big), \quad k_\psi(0) = k_\psi^0
\end{aligned}
\tag{15}
$$

where $\psi^*(t)$ denotes the yaw reference signal. The parametric feedback is used to prevent infinitely increasing when measurements are noisy. This modification is used for all control gains.

To cope with unmodelled actuator dynamics the *shunt* (*parallel feedforward compensator*) is added to the control law (15), cf. [3, 5, 12]. This leads to the following control law:

$$
\begin{aligned}
&\sigma_\psi(t) = \psi(t) - \psi^*(t) + \alpha_\psi \dot{\psi}(t) - v_\psi(t),\\
&u_\psi(t) = -k_\psi(t)\sigma_\psi(t) - \gamma_\sigma \operatorname{sign}(\sigma_\psi(t))\sqrt{|\sigma_\psi(t)|},\\
&\dot{k}_\psi(t) = \gamma_k \sigma_\psi(t)^2 - \alpha_k\big(k_\psi(t) - k_\psi^0\big), \quad k_\psi(0) = k_\psi^0\\
&\dot{v}_\psi(t) = \big(\kappa\tau_\psi(t) - v_\psi(t)\big)\tau^{-1},
\end{aligned}
\tag{16}
$$

where κ and τ are the auxiliary feedback gains and κ is sufficiently small, see [12] for rigorous statements. The following controllers for other angles and positions will be design by analogy with replacing signal $\psi(t)$ and other respective signals. Controller parameters are choosing separately to the each controller.

Linearized pitch (ϑ) and roll (γ) angular motion models may be written as follows: $\ddot{\gamma} = I_x^{-1}\tau_\gamma$ and $\ddot{\vartheta} = I_z^{-1}\tau_\vartheta$. Pitch and roll angles can be obtained by double integration of angular velocities ω_x and ω_z respectively:

$$
\begin{aligned}
\dot{\tilde{x}}_\gamma &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tilde{x}_\gamma + \begin{bmatrix} 0 \\ I_x^{-1} \end{bmatrix} \tau_\gamma, \qquad \tilde{y}_\gamma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tilde{x}_\gamma,\\
\dot{\tilde{x}}_\vartheta &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tilde{x}_\vartheta + \begin{bmatrix} 0 \\ I_z^{-1} \end{bmatrix} \tau_\vartheta, \qquad \tilde{y}_\vartheta = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tilde{x}_\vartheta,
\end{aligned}
\tag{17}
$$

where $\tilde{x}_\gamma = \big[\gamma, \dot{\gamma}\big]^{\mathrm{T}}$ and $\tilde{x}_\vartheta = \big[\vartheta, \dot{\vartheta}\big]^{\mathrm{T}}$.

The passification method can be used since systems (17) fulfill the necessary HMP conditions. The following controllers are used for roll and pitch stabilization as it was shown above for yaw angle (16).

3.3 Adaptive Control Laws for Quadrotor Altitude

As it could be easily derived from (13), quadrotor altitude dynamic may be presented in the following form

$$m\ddot{y} = -mg + \tau_y - \dot{y}A_y, \tag{18}$$

or, in the state-space form with state vector $\tilde{x}_y = \left[y, \dot{y}\right]^{\mathrm{T}}$ as

$$\dot{\tilde{x}}_y = \begin{bmatrix} 0 & 1 \\ 0 & m^{-1}A_y \end{bmatrix} \tilde{x}_y + \begin{bmatrix} 0 \\ m^{-1} \end{bmatrix} \tau_y, \quad \tilde{y}_y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tilde{x}_y. \tag{19}$$

System (19) also satisfies the HMR condition, so the same controller as (16) may be used for altitude stabilization.

3.4 Adaptive Control Laws for Quadrotor Latitude and Longitude

Changing roll and pitch angles allows quadrotor to move along the horizontal plane. Let us describe quadrotor dynamics along $0X$ and $0Z$ as an independent motion from (13):

$$\begin{aligned} m\ddot{x} &= \tau_y(C_\psi S_\vartheta C_\gamma + S_\psi S_\gamma) - V_x A_x, \\ m\ddot{z} &= \tau_y(S_\psi S_\vartheta C_\gamma - C_\psi S_\gamma) - V_z A_z, \end{aligned} \tag{20}$$

Above dynamics equations may be derived taking into account that rotation on yaw angle are done before quadrotor moving in $X0Z$ plane so $\psi = 0$.

Let us use the following *attitude controllers* with pre-rotation on angle ψ^* and coordinate system change:

$$\begin{cases} \gamma^* = -\arcsin \frac{k_{\gamma^*}}{\tau_y}(\cos\psi^*(x - x^*) + \sin\psi^*(z - z^*)), \\ \vartheta^* = \arcsin \frac{k_{\vartheta^*}}{\tau_y \cos\gamma}(\cos\psi^*(z - z^*) - \sin\psi^*(x - x^*)), \end{cases} \tag{21}$$

where x^*, z^* are reference quadrotor position and $k_{\gamma^*}, k_{\vartheta^*}$ are positive coefficients.

With control laws (21) and linearized in the origin model (13), equations describing quadrotor motion in $X0Z$ plane are as follows

$$\begin{cases} k_{\gamma^*}\hat{z}^*(t) = (ms^2 + A_z s + k_{\gamma^*})\hat{z}(t), \\ k_{\vartheta^*}\hat{x}^*(t) = (ms^2 + A_x s + k_{\vartheta^*})\hat{x}(t), \end{cases} \tag{22}$$

where $\hat{z}, \hat{x}$ are stabilization errors in new coordinate system and $\hat{z}^*, \hat{x}^*$ are the reference errors.

In is obviously that transfer function derived from (22) satisfies to HMP conditions with $G = \begin{bmatrix} 1 & \tau_G \end{bmatrix}, \tau_G > 0$. Similarly, we define controllers for latitude and longitude positions.

4 Simulation Results

In the simulations, the model parameters was taken from [7] and some experiments with attitude control based on mentioned above passification method was described in [31]. Controllers coefficient was set the same for every simulation run.

4.1 Stabilization

The most common quadrotor purpose is stabilization in the reference point. Obviously, the reference point is linear position x, z, y and heading angle ψ. Roll and pitch angles are auxiliary to perform transversal motion in 3D space. Let us take reference point $\{x^*, z^*, y^*, \psi^*\}$ as $\{15, 10, 5, \pi/4\}$. Figure 1 shows time histories of latitude, longitude and altitude in the case of constant reference signals. Attitude time histories are shown in Fig. 2.

Auxiliary signals generated by position controllers (21) are shown at fig. 2. Roll and pitch adaptive controllers are performing tracking for generated auxiliary signals. Controller coefficients (including controller (15)) are tuning slightly and fast. Altitude, latitude and longitude controllers coefficient are tuning more substantially are shown in Fig. 3.

4.2 Tracking with noisy measurements

The most common case for quadrotor control is tracking the reference trajectory that may be predefined or manually set by the UAV operator. Let us define the following reference trajectories: $x^*(t) = 8 \sin 0.1t \ (m)$, $z^*(t) = 8 \cos 0.1t \ (m)$, $y^*(t) = 0.1t \ (m)$, $\psi^*(t) = 0.5 \sin t \ (rad)$. All the quadrotor sensors are noisy. Moreover, global positioning are based on GPS with a sufficient inaccuracy. As it was mentioned above, supposed simple adaptive controllers are taking into account noise measurements by using feedback in (15) and the following

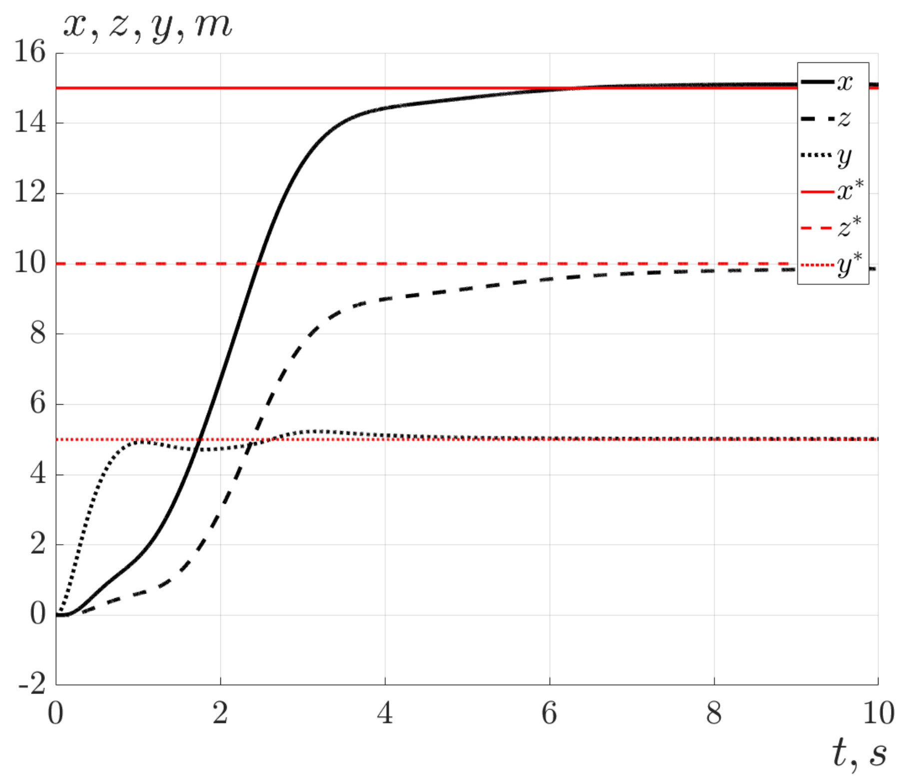

Figure 1: Altitude, latitude and longitude time histories with its constant reference signals.

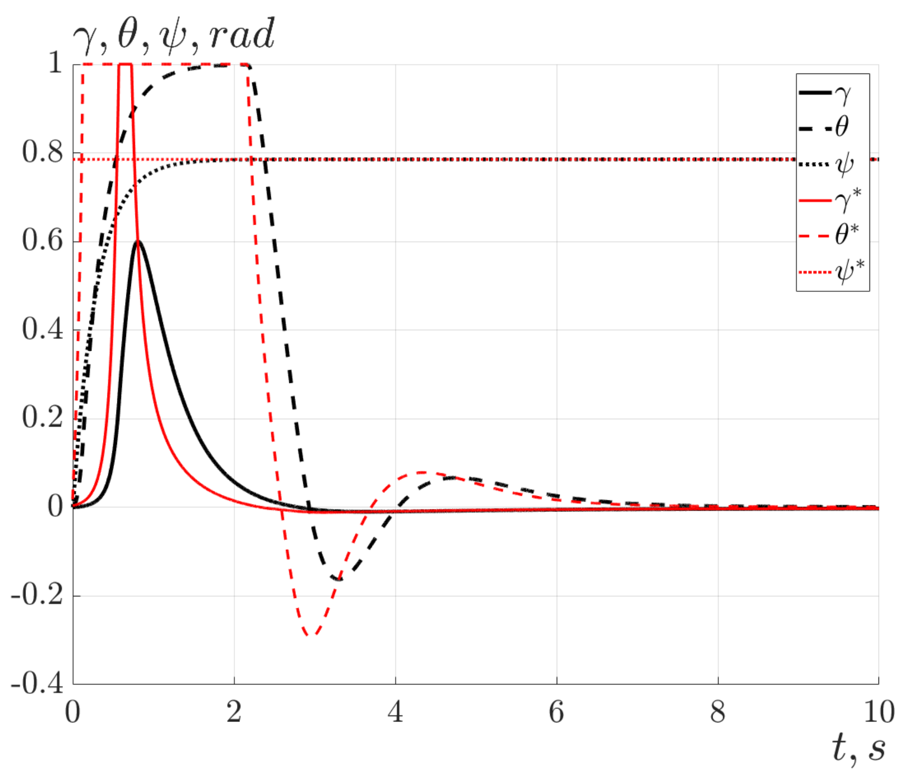

Figure 2: Attitude time histories with its auxiliary signals with constant reference signals.

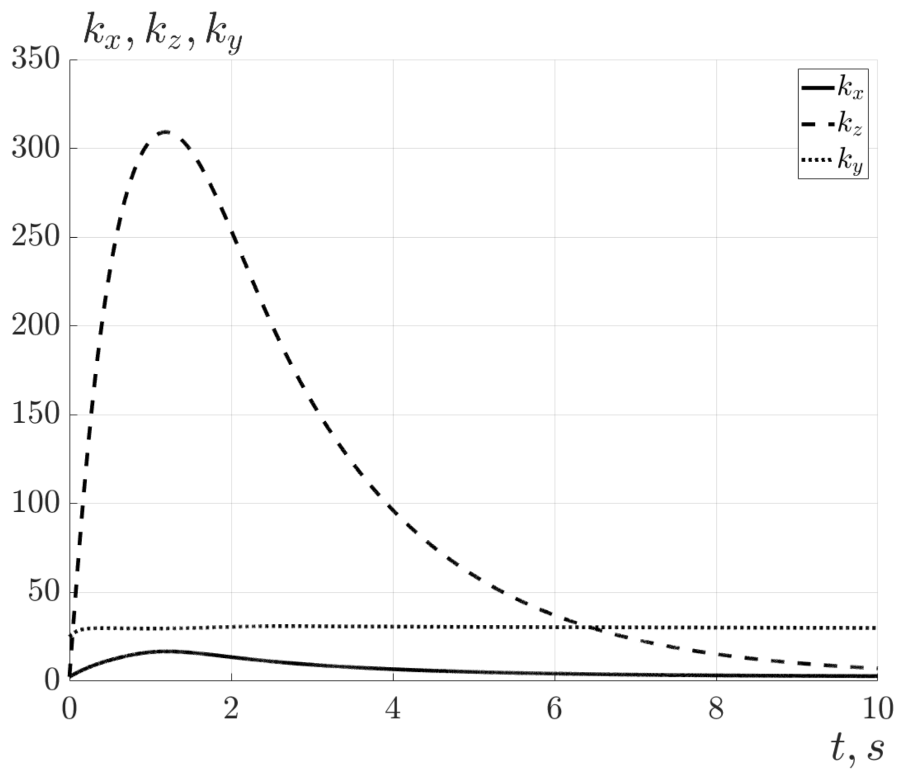

Figure 3: Altitude, latitude and longitude controller coefficients time histories.

controllers. Let us repeat tracking situation and simulate Gaussian noise into IMU (with variance $\sigma_{att}^2 = 0.001$, leads to a noisy attitude position) and into GPS (with variance $\sigma_{pos}^2 = 0.1$, leads to a noisy altitude, latitude and longitude positions). This situation is close to real IMUs measures as it notised in [24]. Noisy measured signals are shown at figs. 4 and 5 by grey lines.

Latitude and longitude reference signals are changing permanently and coefficients in positioning adaptive controllers are also tuning (see fig. 7).

As it could be seen, proposed simple adaptive controllers may be used for quadrotors with noisy sensors and external disturbances.

5 Conclusion

Simple adaptive control technique based on the passification method and the Implicit Reference Model approach is used for design of the adaptive controller for every quadrotor position: attitude, altitude, latitude and longitude. The simulation study has been preformed basing on previous experimental studies and quadrotor mathematical models. The simulations demonstrate good performance quality and high adaptation rate of the SAC system for every controlling

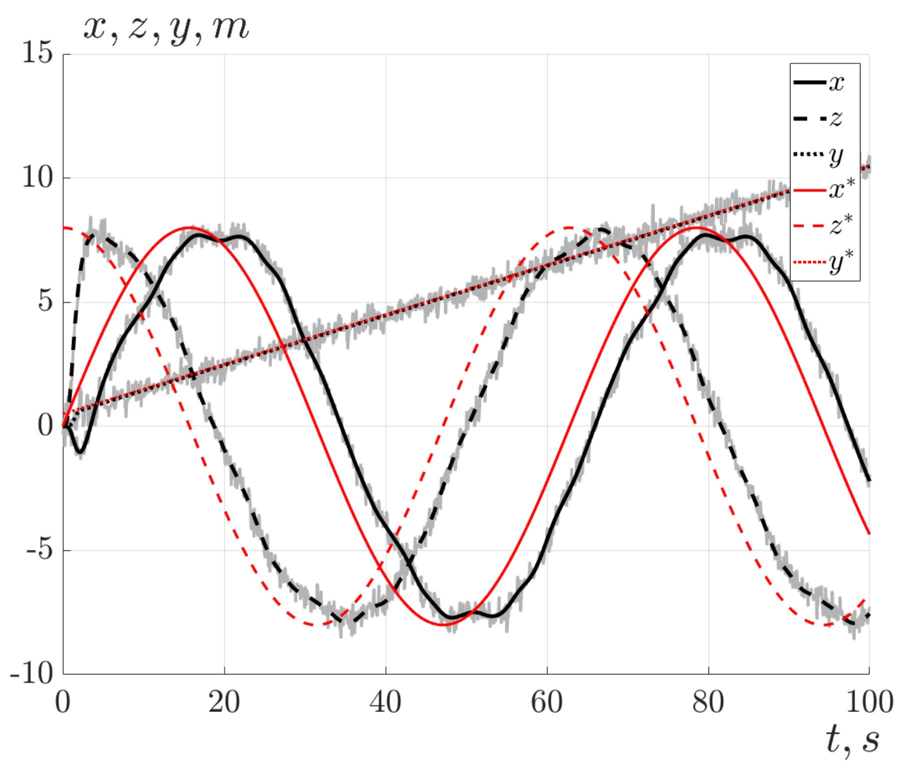

Figure 4: Altitude, latitude and longitude time histories with its constant reference signals and noisy measurements.

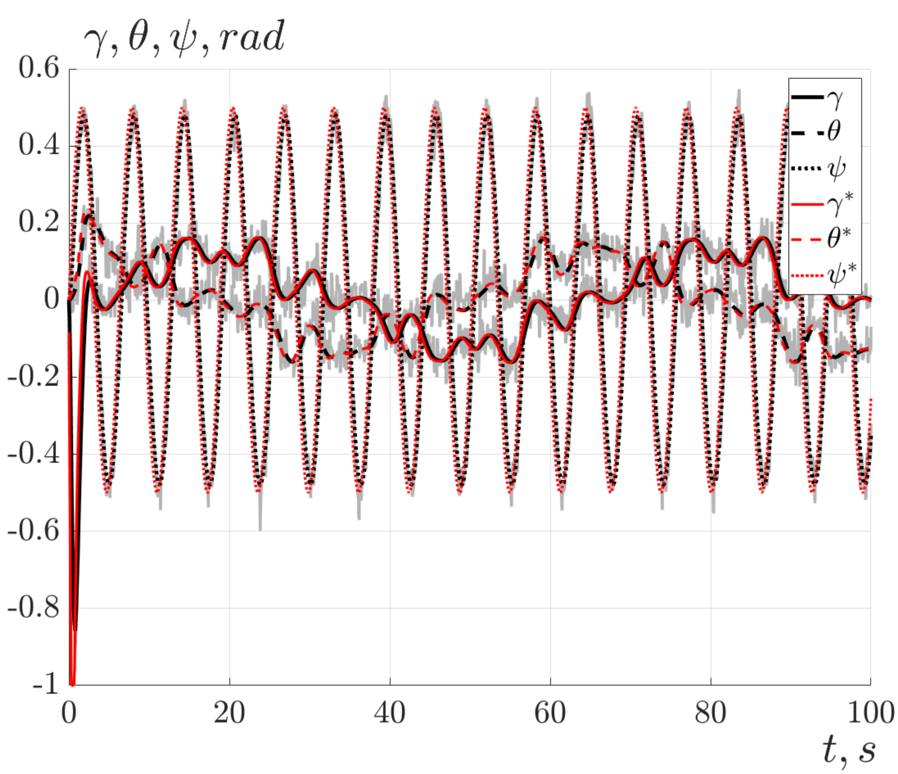

Figure 5: Attitude time histories with its auxiliary signals with constant reference signals and noisy measurements.

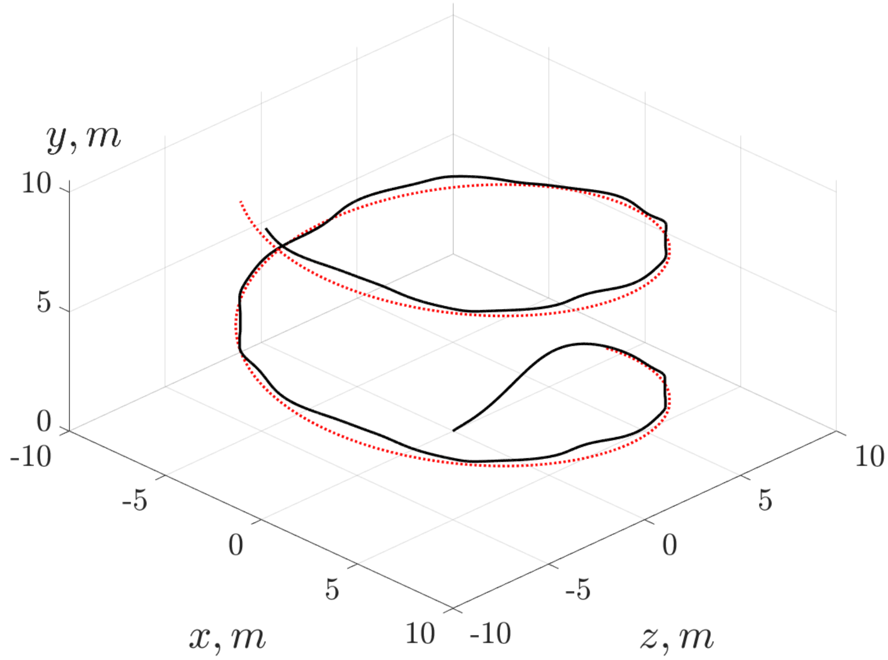

Figure 6: Quadrotor trajectory while stabilizing in reference point.

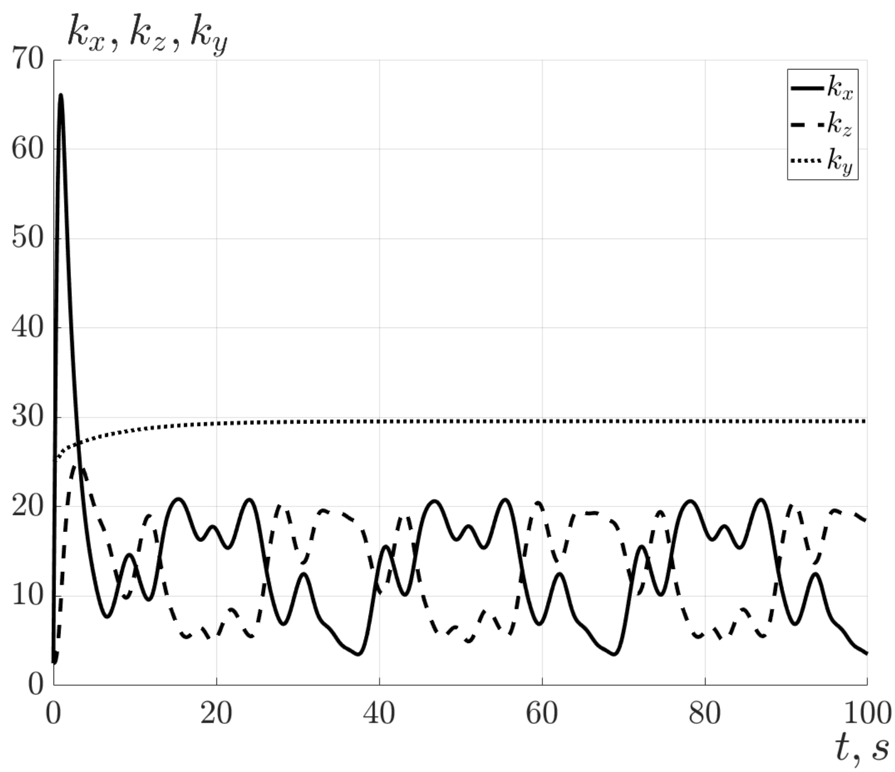

Figure 7: Altitude, latitude and longitude controller coefficients time histories.

signals. In the future it is planned to extend in-flight testing and implement full control of based on passification method for the real quadrotor.

Acknowledgment

This work was performed the Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences (IPME RAS) under Goscontract A19-119120290136-9, and supported in part by the Russian Foundation for Basic Research (grant № 18-38-20037) and in part by the Ministry of Science and Higher Education of Russian Federation (grant № FZWF-2020-0015).

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