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Optimal Design of Model Following Control with Genetic Algorithm

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Abstract.

In this paper, an integral-type robust model following control for plants with uncertain parameters is introduced. An genetic algorithm is also designed for obtaining the control gains. Finally, some numerical examples are provided to illustrate the validity and efficiency of the proposed method.

Keywords: genetic algorithm, model following control, uncertain systems.

1 Introduction

Model following control system are designed to make the outputs of the plant having desired performance. The general steps of control system design are : the first is to model the plants and then control schemes are designed based on the reference model. However, for a real controlled plant there exist some difficulties to construct an exact plant model which necessitates the accurate system parameters. Some parameters are time-varying and functions of the system states. When we design a control system, these uncertainties have to be considered.

There are many research reports on designing robust controller for the uncertainty system, for example, the method by using H^∞/μ control theory [1,2], the method based on robust control method[3,4], the method of gain scheduled controller [5,6]. Kimura and Watanabe [1, 2] assume the uncertainty of the transfer function of the controlled system being $\Delta(s)$, and the necessary condition is $\Delta(s) \leq \gamma(s)$. The problem of this method is that the selection of $\gamma(s)$ requires the character information of uncertainty. In literature [3,4], the uncertainties of parameters of the controlled system are considered as perturbations. When the perturbation is stable a robust controller is presented by using gain scheduled control scheme, however, if the system is not completely observable, state observers have to be employed. The stability of the control system can not be guaranteed by gain scheduled controller.

In this paper, an integral type robust model following control scheme with two degrees of freedom for plants with uncertain parameters is presented. Simulation results are given to illustrate the validity and efficiency of the proposed method.

2 Problem Description

Consider the following SISO systems described by n order controllable canonical form

$\sum_p$:

$$\begin{cases} \dot{x}_p(t) &= A_p x_p(t) + b u_p(t) \\ y_p(t) &= c_p^T x_p(t) \end{cases} \tag{1}$$

$\sum_m$:

$$\begin{cases} \dot{x}_m(t) &= A_m x_m(t) + b u_m \\ y_m(t) &= c_m^T x_m(t) \end{cases} \tag{2}$$

where $\sum_p$ is the plant, $\sum_m$ is the stable reference model, and

$$A_p = N + b\{a_p + \Delta a_p\}^T$$
$$A_m = N + b a_m^T$$
$$b = (0, 0, \cdots, 0, 1)^T \in R^n$$

$$N = \left(\begin{array}{c|c} & I_{n-1} \\ 0 & \overline{\phantom{I_{n-1}}} \\ & 0 \end{array} \right) \in R^{n \times n}$$

In above equations, Δa_p is a term of uncertainty with known upper and lower boundaries. a_m^T is the parameter of the reference model, u_m is a step signal. Parameters a_p and c_p are known, and $x_p(t)$ are observable.

The stable integral type interacter polynomial of the plant is considered to be

$$\sigma(s) = s^\nu + \mu_\nu s^{\nu-1} + \ldots + \mu_1 + \frac{\mu_0}{s}$$

where $\mu = (\mu_0, \cdots, \mu_\nu) \in R^{\nu+1}$ and ν is the relative degree of the system.

In order to obtain robust model following performance when there exist time-varying parameters, an integral compensator is necessary. Therefore, the control input in equation (1) is assumed as equation (3), and the following error is given in equation (4).

$$\begin{aligned} u_p(t) = &-k_p^T x_p(t) + k_m^T x_m(t) + m u_m(t) \\ &-f_p^T \int x_p(t)dt + f_m^T \int x_m(t)dt \end{aligned} \tag{3}$$

$$e(t) = y_p(t) - y_m(t) \tag{4}$$

where k_p, f_p, k_m and $f_m$are the gains of feedback and feedforward signals, respectively.

The purpose of this paper is to present an integral type linear control scheme with fixed gains so that $e(t) \to 0$ for any input u_m.

When $\Delta a_p = 0$, the following results are given in [7]

$$k_p^T = \frac{1}{w_p} c_p^T \bar{\sigma}(A_p) \tag{5}$$

$$f_p^T = \frac{\mu_0}{w_p} c_1^T \tag{6}$$

$$\frac{k_m^T}{m} = \frac{1}{w_m} c_m^T \bar{\sigma}(A_m) \tag{7}$$

$$\frac{f_m^T}{m} = \frac{\mu_0}{w_m} c_m^T \tag{8}$$

where $w_p = c_p^T A_p^{\nu-1} b, w_m = c_m^T A_m^{\nu-1} b, m = \frac{w_m}{w_p}$,

and $\bar{\sigma}(s) = s^\nu + \mu_\nu s_1^{\nu-1} + \cdots + \mu_1$

Based on the above results, we introduce the following evaluation function

$$CP = \int_0^T \{e^T(t) Q e(t) + u_p(t)^T R u_p(t)\} dt \tag{9}$$

where T is the time of one test control. However, CP is essentially a stochastic variable because Δa_p is indeed stochastic.

According to the evaluation function, in the following section a robust model following scheme for $\Delta a_p \neq 0$ is presented based on genetic algorithm.

3 Design of controller with GA

In this section a genetic algorithm for the design of the robust model following controller is developed. The following problems will be discussed on : initialization process, evaluation function, selection operation, crossover and mutation operations.

3.1 Initialization Process

We suppose the initial number of generation is 0 (i.e gen = 0), and the end number of generation is GEN, and assume following vector

$$V = \begin{pmatrix} \mu_0 & \mu_1 & \cdots & \mu_\nu \end{pmatrix}$$

being the chromosome to represent the optimal robust solution of the model following system, and define an integer *pop_size* as the number of chromosomes. *pop_size* chromosomes will be randomly initialized by the following steps:

Step 1. Determine an interior point, denoted by V_0, in the constraint set.

Step 2. Select randomly a direction d in $R^{2\times n}$ and define a chromosome V as $V_0 + M \cdot d$ if it is feasible, otherwise, we set M as a random number in $[0, M]$ until $V_0 + M \cdot d$ is feasible, where M is a large positive number which ensures that all the genetic operators are probabilistically complete for the feasible solutions.

Step 3. Repeat Step 2 *pop_size* times and produce *pop_size* initial feasible solutions.

3.2 Stability and test control

Step 1. According to Hurwitz criterion the interacter polynomial is checked, if it is stable then go to Step 2, otherwise, go to Step 3.

Step 2. According to equations (5)-(8) control gains are calculated, test control is executed and then go to section 3.3.

Step 3. Test control is not executed because the stability of the system is unknown, then go to section 3.4.

3.3 Evaluation Function

According to equation (9) evaluation function $CP(V_k)$ is calculated, where $k = 1, 2, \cdots, pop\_size$, *pop_size* are the numbers of chromosome. The bigger the value of evaluation function is, the stronger the adaptation degree of the corresponding chromosome becomes.

3.4 Selection Operation

The selection process is based on spinning the roulette wheel *pop_size* times, each time we select a single chromosome for a new population in the following way:

Step 1. Calculate a cumulative probability a_i for each chromosome V_i, $(i = 1, 2, \cdots, pop\_size)$.

Step 2. Generate a random real number r in $[0, 1]$.

Step 3. If $r \leq a_1$, then select the first chromosome V_1; otherwise select the i-th chromosome V_i $(2 \leq i \leq pop\_size)$ such that $a_{i-1} < r \leq a_i$.

Step 4. Repeat Steps 2 and 3 *pop_size* times and obtain *pop_size* copies of chromosomes.

In this process, the best chromosomes get more copies, the average stay even, and the worst die off.

3.5 Crossover Operation

We define a parameter P_c of a genetic process as the probability of crossover. This probability gives us the expected number $P_c \cdot pop\_size$ of chromosomes which undergo the crossover operation.

Firstly we generate a random real number r in $[0, 1]$; secondly, we select the given chromosome for crossover if $r < P_c$. Repeat this operation *pop_size* times and produce $P_c \cdot pop\_size$ parents, averagely. For each pair of parents (vectors V_1 and V_2), the crossover operator on V_1 and V_2 will produce two children as

$$\begin{aligned} V_1' &= \lambda_1 \cdot V_1 + \lambda_2 \cdot V_2, \\ V_2' &= \lambda_2 \cdot V_1 + \lambda_1 \cdot V_2 \end{aligned}$$

where λ_1, $\lambda_2 \geq 0$ and $\lambda_1 + \lambda_2 = 1$.

Since the constraint set is convex, this arithmetical crossover operation ensures that both children are feasible if both parents are.

3.6 Mutation Operation

We define a parameter P_m of a genetic process as the probability of mutation. This probability gives us the expected number $P_m \cdot pop\_size$ of chromosomes which undergo the mutation operation.

Generating a random real number r in $[0, 1]$, we select the given chromosome for mutation if $r < P_m$. Let a parent for mutation, denoted by a vector

$$V = \begin{pmatrix} \mu_0 & \mu_1 & \cdots & \mu_\nu \end{pmatrix}$$

be selected. Select randomly a direction d in $R^{2\times n}$ and define a chromosome V as $V_0 + M \cdot d$ if it is feasible, otherwise, we set M as a random number in $[0, M]$

until $V_0 + M \cdot d$ is feasible, where M is a large positive number defined in the initialization process.

gen $\leftarrow$ gen + 1

If gen $\leq$ GEN then go to section 3.2, otherwise, simulation will end and the gain corresponding to the chromosome with maximum adaptation degree is used as a control gain.

Repeat above process for all chromosomes.

Following selection, crossover and mutation, the new population is ready for its next evaluation. The algorithm will terminate after a given number of cyclic repetitions of the above steps.

4 Numerical Examples

Here we will illustrate the effectiveness of proposed genetic algorithm for the optimal design of model following control by some numerical examples. Computer simulations are executed on NEC EWS4800/210II workstation with the following parameters: the population size is 30, the probability of crossover P_c is 0.2, the probability of mutation P_m is 0.4.

A second order plant is described as follows

$$\dot{x}_p(t) = \begin{bmatrix} 0 & 1 \\ -0.5 + \Delta a_{p1} & -1 + \Delta a_{p2} \end{bmatrix} x_p(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_p(t)$$

$$y_p(t) = [1 \quad 0]\, x_p(t)$$

where $x_p(t) = [x_{p1}(t), x_{p2}(t)]^T$ is the state vector of plant and $u_p(t)$ is the control signal. Δa_{p1} and Δa_{p2} are uniformly distributed variables on the interval [-0.2, 0.2] and [-0.4, 0.4].

The reference model is described as follows

$$\dot{x}_m(t) = \begin{bmatrix} 0 & 1 \\ -2.5 & -3 \end{bmatrix} x_m(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_m$$

$$y_m(t) = [1 \quad 0]\, x_m(t)$$

where $x_m(t) = [x_{m1}(t), x_{m2}(t)]^T$ is the state vector of reference model. The initial state is

$$x_p(0) = (0.6, 0)^T, x_m(0) = (0, 0)^T.$$

The input of reference model is $u_m = 4$.

The purpose of the control is that the output of the plant with parameters uncertainties can follow the output of the reference model robustly and fast, and the control $u_p(t)$ is oppressed within a range not to keep away into big signals. Here based on the method stated in section 3, the parameter μ_i of the interacter polynomial is determined. In this simulation study the test execution time T is 20 second, the weights of the evaluation function CP are Q=3 and R=0.5, the end generation number is GEN =100.

The programs for the proposed genetic algorithm are written in C language. We use it to solve above numerical example. In this example, $\nu = 2$, we restrict the parameters in the following set

$$\left\{(\mu_2, \mu_1, \mu_0) \;\middle|\; 0 \le \mu_i \le 30, \quad i = 0, 1, 2\right\}$$

which is clearly convex.

The following parameters

$$(\mu_2, \mu_1, \mu_0) = (3.77, 2.94, 3.19)$$

are used and the corresponding control gains are $k_p^T = (3.27, 1.94), k_m^T = (-3.52, 4.44)$, $f_p^T = (3.19, 0)$ and $f_m^T = (0.11, 0)$. So, the following control

$$u_p(t) = -(3.27, 1.94)x_p(t) + (-3.52, 4.44)x_m(t)$$
$$+3.19 \int \{x_{m1}(t) - x_{p1}(t)\}dt$$

is obtained. The total CPU time of the NEC EWS4800/210II workstation is 786.5 seconds.

We have figured out the results to demonstrate the evolutionary process of the performance index by Fifure 1. Using the proposed GA approach, the optimal solution is obtained to take a minimum value 182 at 10 generations.

The plant and reference model output signals obtained by genetic algorithm are shown in Figure 2∼Figure 5. In Figure 2 the output of the plant is given when there is no parameter uncertainty, that is, Δa_{p1} and Δa_{p2} are zero. Figure 3 gives the output $y_p(t)$ when $\Delta a_{p1} = 0, \Delta a_{p2} \neq 0$. Figure 4 is the case when $\Delta a_{p1} \neq 0, \Delta a_{p2} = 0$ and Figure 5 gives the output of $y_p(t)$ when all of $\Delta a_{p1}, \Delta a_{p2}$ are not zero. From these simulation results it can be concluded that the output of plant can fast and robustly follow the output of reference in spite of the existence of plant parameter uncertainties and nonlinear factors by using the designed control $u_p(t)$.

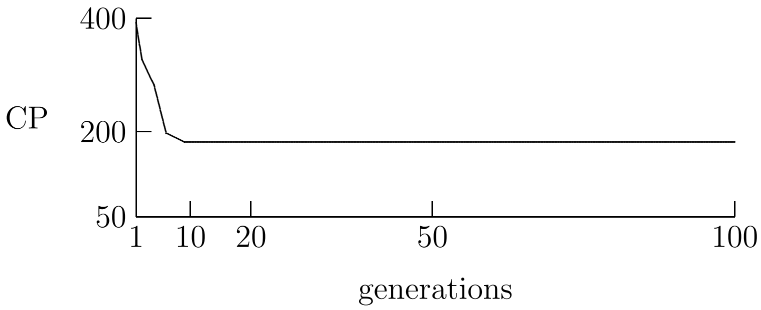

Figure 1: Evolutionary process by GA

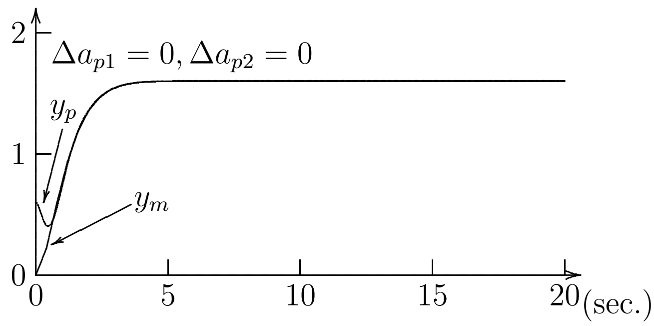

Figure 2: Plant and reference model output signals

5 Conclusion

In this paper, based on GA a integral type robust model following control scheme for plants with parameters uncertainties is presented. According to stability criterion the stability of the system can be determined, and the control gains can be efficiently obtained. Simulation results show that system has satisfactory following properties.

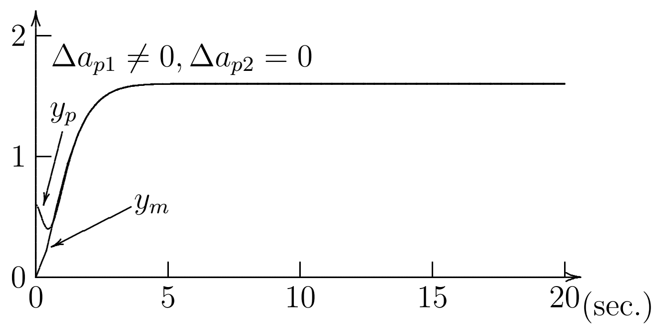

Figure 3: Plant and reference model output signals

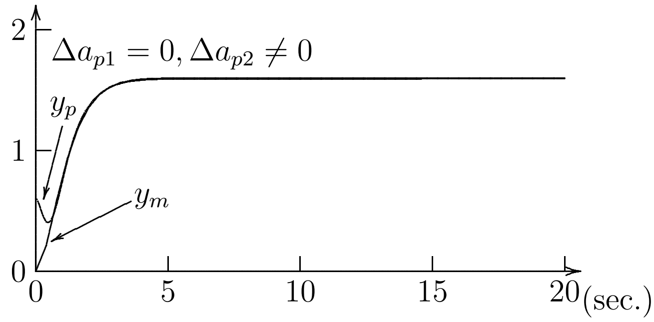

Figure 4: Plant and reference model output signals

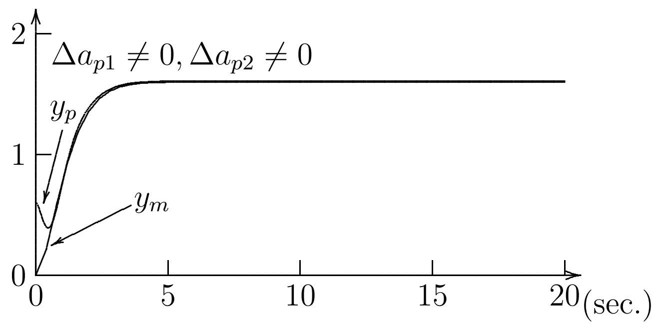

Figure 5: Plant and reference model output signals

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