DIFFERENTIAL EQUATIONS
AND
CONTROL PROCESSES
N 3, 1999
Electronic Journal,
reg. N P23275 at 07.03.97

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Applications to physics, electrotechnics, and electronics

NONLINEAR BOUNDARY VALUE PROBLEMS DESCRIBING MOBILE CARRIER TRANSPORT IN SEMICONDUCTOR DEVICES

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Abstract.

The present paper describes mobile carrier transport in semiconductor devices with constant densities of ionized impurities. For this purpose we use one-dimensional system of partial differential equations.The work considers the proofs of: global existence of solutions of systems of such kind, their bifurcations and their stability under corresponding assumptions.

Key words: Nonlinear boundary value problem, asymptotic behaviour of solutions, semiconductors, carrier transport, constant densities of ionized impurities, interior transition layer phenomena.

Classification: 35Q20, 35D05, 35B40.

1. Introduction

This article inteds to extend the theoryof L.Recke [1]. In his paper he considers a simple mathematical model decribing mobile carrier transport in semiconductor devices. Two functions $E(x)$ and $n(x)$ describle electric feld strength and density of mobile electrons and satisty under $0 < x < 1$ the system

$$(D(|E|)(n' + nE))' = 0, \tag{1}$$

$$E' = f - n,$$

and boudary conditions

$$E(0) = E(1) = E_0, \qquad D(|E(0)|)\left(n'(0) + n(0)E(0)\right) = j_0. \tag{2}$$

Here the constant $f > 0$ represents a homogeneons density of ionized impurutues, $D(|E|)$ is the diffusion coefficient, j_0 is the electron currien density for $x = 0$. For sake of definiteness we shall presume $E_0 > 0$. The article [1] proposes

$$j_0 = D(E_0)E_0 f. \tag{3}$$

Under such conditions the problem (1)–(3) has a trivial solution $E(x) = E_0$, $n(x) = f$. If $K(E_0) < 0$ (where $K(E_0) = 1 + D'(E_0)E_0D^{-1}(E_0)$) then there exists a denumerable set of points $f_k(E_0) = -K(E_0)^{-1}\left(E_0^2/4 + \pi^2 k^2\right)$, $k = 1, 2, \ldots$, in the neighborhood of which appear small bifurcational solutions of the problem (1)–(3) [1]. In this paper we prove that the condition $K(E_0) < 0$ implies that the diffusion coefficient D as a function of fieldstrength has N–shaped form and contains an interval (E_1, E_2), in which this function has a negative derivative and $D(E_0) + E_0D'(E_0) < 0$, i.e. so called condition of negative differential conductivity (NDC) is valid. In section 1 we prove that NDC–condition is necessary and sufficient for the existance of bifurcational solutions of the problem (1)–(3). Then in section 2 the extendability of all bifurcational solutions on the parameter f is demonstrated. The corresponding bifurcational problem can be considered as an eigenvalue nonlinear problem; in section 3 we discuss forms of eigenfunctions of this problem. It appears that asymptotic behavior of bifurcational solutions under large values of parameter f depends essentially on the parameter E_0. In section 4 we prove that a unique point E_0^* exists on the interval (E_1, E_2) such that there arise so called interior transition layer phenomena [2,3] in the problem (1)–(3). If $E_0 \in (E_1, E_2)$, but $E_0 \neq E_0^*$, then asymptotic behavior of bifurcational solutions has a completely different nature. In section 5 the existence and uniqueness of solutions of nonstationary initial–boundaru value problem for (1)–(3 for every $t > 0$ is discussed; this theory is applied to investigate stability and instability of bifurcational solutions. It appears that the stability of the first (positive) and the second (negative) eigenfunctions depends also on the parameter E_0. In section 6 ti is proved that:

if $E_0 = E_0^*$, then the pair eigenfunctions are stabile; if $E_1 < E_0 < E_0^*$, then only the negative function is stable, and if $E_0^* < E_0 < E_2$ or vise versa, only positive one is stable. Other eigenfunctions are unstabile for any $E_0 \in (E_1, E_2)$ for those values of the parameter f for which they exist. In the last section 7 the existence of parabolic travelling wave for $E_0 = E_0^*$ and its stability for sufficiently large f are proved.

2. Existence of bifurcational solutions dependent on the parameter E_0

The problem(1)–(3) is equivalent to the following boundary value problem

$$\begin{gathered} E'' + E'E = fH(E, E_0), \quad 0 < x < 1, \\ E(0) = E(1) = E_0, \end{gathered} \tag{4}$$

where $H(E, E_0) = E - E_0 D(E_0) D^{-1}(|E|)$.The last problem has a trivial solution $E(x) = E_0$ for any f. Let the diffusion coefficient satisfy the conditions:

1) $D(y) \in C^{(2)}(R_+), \quad D\colon R_+ \to R_+$;

2) $D(y)$ has a unique local maximum and a unique point of inflection for $y > 0$;

3) $\lim\limits_{y \to +\infty} D(y) = D_0 > 0$.

Let $E(x) = E_0 + u(x)$ and $g(u) = D(E_0)D^{-1}(|E_0 + u|) - 1$. The function $g(u)$ is continiously differentiable for $u \neq -E_0$ and

$$0 < g_i = \sup_u \left| g^{(i)}(u) \right| < +\infty, \quad i = 0, 1.$$

Proposition 2.1 *If the inequality $f < \dfrac{\pi^2}{E_0 g_1}$, is valid, then the problem (4) has a unique (trivial) solution.*

Let us write the problem (4) in the form

$$\begin{gathered} -u'' - E_0 u' + fu - u'u = E_0 f g(u); \\ u(0) = u(1) = 0, \end{gathered} \tag{5}$$

and let u be a nontrivial solution of this problem. Multiplying the second equation (5) by $u(x)$ and integrating by parts we obtain

$$\int\limits_0^1 u'^2(x)dx \leq E_0 f g_1 \int\limits_0^1 u^2(x)dx,$$

and since

$$\pi^2 \int_0^1 u^2(x)dx \le \int_0^1 u'^2(x)dx, \tag{6}$$

then the estimate $f \ge \dfrac{\pi^2}{E_0 g_1}$ holds, which gives the proposition 1.1.

Let $C_0^{(2)}([0,1])$ be the space of functions $u(x)$ which are continious on$[0,1]$, have the second continious derivatives on $(0,1)$ and for $u(x)$ the conditions $u(0) = u(1) = 0$ are fulfilled; let $C([0,1])$ be the space of continious functions on $[0,1]$. We shall consider the problem (5) as a nonlinear eigenvalues problem:

$$\begin{gathered} Lu + fK(E_0)u + N(E_0, f, u) = 0; \\ u(0) = u(1) = 0, \end{gathered} \tag{7}$$

where $Lu = -u'' - E_0 u'$ is the linear operator mapping from $X = C_0^{(2)}([0,1])$ into $Y = C([0,1])$, $K(E_0) = 1 + D'(E_0)E_0 D^{-1}(E_0)$, and $N(E_0, f, u) = -u'u - E_0 f \left(\dfrac{D(E_0)}{D(|E_0 + u|)} + \dfrac{D'(E_0)u}{D(E_0)} - 1 \right)$ is the nonlinear one mapping from $R^2 \times X$ into Y. By S we denote the closure of the set of all nontrivial solutions $(f, u) \in R \times X$ to (7) with $u \ne 0$, and let S_k be the maximal connected component of S containing $(f_k, 0)$, ΘΛΞ $f_k = -K(E_0)^{-1}\left(E_0^2/4 + \pi^2 k^2\right)$, $k = 1, 2, ...$

Theorem 1.1 *Suppose $K(E_0) < 0$. Then the following holds:*

(i) S_k is unbounded;

(ii) suppose $(f, u) \in S_k$ and $u \not\equiv 0$. Then $u(x)$ has exactly $(k+1)$ zeros in $[0,1]$, and all zeros are simple;

(iii) for all $k \in \mathbf{N}$ there exist a constant $s_k > 0$, a neighbourhood $U_k \subset R \times X$ of $(f_k, 0)$ and two C^1-mappings $\hat{f}_k\colon (-s_k, s_k) \to R$, $\hat{u}_k\colon (-s_k, s_k) \to X$ such that $\hat{f}_k(s) = f_k + O(s)$, $\hat{u}_k(s) = su_k(x) + O(s^2)$ for $s \to 0$ and $S \cap U_k = \left\{(\hat{f}_k(s), \hat{u}_k(s))\colon |s| < s_k\right\}$, where $u_k(x) = e^{-E_0 x/2}\sin(\pi k x)$.

These solutions are called bifurcational ones [4].

It is fairly evident that the condition $K(E_0) < 0$ is equvalent to the condition of negative differentional conductivity (NDC) (see Introduction).

Proposition 2.2 ([5]) *Let $D(|E|)$ satiafy the NDC. Then there exists a unique E_0^* such, that*

a) $0 < G(E_{\min}) < G(E_0^*) < G(E_{\max})$, where $E_{\max}$, $E_{\min}$ – the local extrema of function $G(E) = ED(E)$ for $E > 0$;

b) the derivative $G'(E_0) < 0$ *for* $E_0 \in (E_{\max}, E_{\min})$ *which is equvalent to the condition* $K(E_0) < 0$*;*

c) the equation $H(E, E_0) = 0$ *for* $E_0 \in (E_{\max}, E_{\min})$ *has only three positive solutions* $0 < E_1(E_0) < E_0 < E_2(E_0)$*, moreover* $H'_E(E_i(E_0), E_0) > 0$*,* $i = 1, 2$*;*

d) $$\int\limits_{E_1(E_0^*)}^{E} H(s, E_0^*) ds \begin{cases} > 0, & \text{for } E \in (E_1(E_0^*), E_2(E_0^*)), \\ = 0, & \text{for } E = E_2(E_0^*). \end{cases}$$

Let us analyse for which values of the parameter E_0 nontrivial solutions of problem (4) exist.

Proposition 2.3 *If* $0 < E_0 \leq E_{\max}$ *or* $E_0 \geq E_{\min}$ *thenthe problem (4) has only trivial solution* $E(x) = E_0$*. If* $E_0 \in (E_{\max}, E_{\min})$ *then the problem (4) has nontrivial (bifurcaational) solutions.*

If $E_0 \in (E_{\max}, E_{\min})$, then $G'(E_0) < 0$, i.e. $K(E_0) < 0$ and from theorem 1.1 the problem (4) has nontrivial solution. If $0 < E_0 \leq E_{\max}$ or $E_0 \geq E_{\min}$ then $K(E_0) \geq 0$. Let us consider the problem (7). Further we prove that if $K(E_0) \geq 0$ then thia problem has no small nontrivial solutions; from this and from Rabinowitz's results (see theorem 2.3 [6]) it follows that the problem (7) can not have any nontrivial solutions. The problem (7) can be linearized in the neighbourhood of a zero-solution

$$Lu = -fK(E_0)u;$$

$$u(0) = u(1) = 0.$$

Since the last form of the problem has only a zero-solution, then the problem (7) has no nontrivial solutions ■ .

3. Extendability of bifurcational solutions on parameter f

We show in this section that every bifurcational solution is extendable on the parameter $f > f_k$, where $f_k = -K(E_0)^{-1}\left(E_0^2/4 + \pi^2 k^2\right)$, $k = 1, 2, \ldots$. Let us prove the following proposition.

Proposition 3.1 *Let* $E_0 \in (E_{\max}, E_{\min})$*. Then there exists such a positive continius fuiction* $\varphi(f)\colon R_+ \to R_+$*, that for any solution* (u, f) *of the problem (7) the inequality:*

$$\|u\|_X(f) \leq \varphi(f) \tag{8}$$

is fulfilled.

Proof. From the form of differential equation and the inequality (6) we have an estimate

$$\|u'\|_{L_2} \leq \pi^{-1} E_0 f g_0, \tag{9}$$

This estimate gives an analogous estimate in the norm $C^0([0,1])$. The estimate of $u''(x)$ is based on (7) and inequalities (6) and (9). As a result we have

$$\|u''\|_{L_2} \leq c_1 f^2 + c_2 f^3 + c_3 f^4,$$

where constants c_i depend only on E_0 and g_0. The same estimate is valid for $u(x)$ in the norm $C^{(1)}([0,1])$. To estimate the uniform norm of $u''(x)$ the equation (7) can be used; it gives the estimate (8).

Now return to the problem (7). If $E_0 \in (E_{\max}, E_{\min})$, then form the proposition

a) of Theorem 1.1 and proposition 3.1 it follows that the bifurcational solutions which were obtained in the proposition

b) of theorem 1.1 are extendable on the parametr f for any $f > f_k$, $k = 1, 2, ...$

To synthesize these results we denote U_k^+ to be set of $u(x) \in X$, which have $(k+1)$ simple zeros and $\operatorname*{sign}_{x \to 0+0} u(x) = 1$; let $U_k^- = -U_k^+$, $k \in N$.

Theorem 3.1 *Given* $E_0 \in (E_{\max}, E_{\min})$*. Then for every* $k \in N$*, every* $\nu = +$ *or* $-$ *and for every* $f > f_k$ *there exists at least one solution* $u(x)$ *of boundary value problem (7) such that* $u \in U_k^\nu$*.*

In further text the functions $u_k^\nu(x) \in U_k^\nu$ will be called eigenfunctions of the nonlinear operator of the problem (7).

4. Forms of eigenfunctions.

To investigate the forms of these eigenfunctions let the problem (4) be reformulated as:

$$\begin{gathered} -E'' - E'E = f(G(E_0) - G(E))D^{-1}(|E|), \\ E(0) = E(1) = E_0. \end{gathered} \tag{10}$$

If $E_0 \in (E_{\max}, E_{\min})$ and (f, E) is a nontrivial solution of the problem (10) the for each $x \in [0,1]$ the inequality $E_1(E_0) < E(x) < E_2(E_0)$ is fulfilled. In this inequality $E_i(E_0)$ $(i = 1, 2)$ is the solution of the equation $H(E, E_0) = 0$, which was mentioned in the proposition 2.2, $i = 1, 2$. This result follows from the fast that constants E_0, $E_1(E_0)$ and $E_2(E_0)$ are the solutions of differential equation (10).

The solutions $E(x)$ of the problem (10) will be called a positive one if a corresponding function $u(x) = E(x) - E_0$ is positive on $(0, 1)$.

Let $E(x)$ be the positive solution of the problem (10). Let us prove that $E(x)$ has only one maximum. If it is not so then there exists such $x_0 \in (0, 1)$ that $E(x_0)$ is a localminimum. However that given

$$0 \geq -E''(x_0) = f(G(E_0) - G(E(x_0)))D^{-1}(E(x_0)) > 0,$$

wich is impossible.

Theorem 2.1 shows that each solution from S_k has $(k + 1)$ simple zeros on$[0, 1]$; similar reasoning gives next result.

Proposition 4.1 *Let $E(x) = u(x) + E_0$ be such a solution of the boudary value problem (10) that $(f, u) \in S_k$, $k = 1, 2, ...$ Then:*

a) if $k = 2n$, then $E(x)$ has n maxima and n minimuma;

b) if $k = 2n + 1$ then the numbers of minima and maxima differ by 1.

5. Asymptotic behaviour of the eigenfunctions for large values of parameter f

The case of large concentrations of ionized impurities is of gread interest for physical application. Mathematically this fact can be associated with asymptotical behaviour of the eigenfunctions for large values of parameter f. Let the problem (4) be formulatrd as:

$$\begin{gathered} \varepsilon E'' + \varepsilon E'E = H(E, E_0), \\ E(0) = E(1) = E_0, \end{gathered} \tag{11}$$

where $\varepsilon = f^{-1}$, $H(E, E_0) = E - E_0 D(E_0) D^{-1}(|E|)$.

Let $u_k^\nu(x, f)$ be the bifurcational solutions of the problem (7), $f > f_k$, $k = 1, 2, ...$, $\nu = +, -$. Then $E_k^\nu(x, \varepsilon) = u_k^\nu(x, \varepsilon^{-1}) + E_0$ are the solutions of the problem (11), which are defined for $0 < \varepsilon < \varepsilon_k$, where $\varepsilon_k = f_k^{-1}$. We shall call these the eigenfunctions of the problem (11). Let $E_0 \in (E_{\max}, E_{\min})$ and $E_i(E_0)$, $i = 1, 2$ be two positive solutions of the equation $H(E, E_0) = 0$ from the proposition 2.2. The main results of this section are the next three theorems.

Theorem 5.1 *Given $E_0 \in (E_{\max}, E_0^*)$. The problem (11) has families of solutions $E_1^-(x,\varepsilon)$, $E_k^\pm(x,\varepsilon), k = 2,3,...,$ defined for sufficiently small ε; these families have the following properties:*

$\lim\limits_{\varepsilon\to 0+0} E_1^-(x,\varepsilon) = E_1(E_0)$ *uniformly for every compact set from* $(0,1)$*;*

$\lim\limits_{\varepsilon\to 0+0} E_k^\pm(x,\varepsilon) = E_1(E_0)$ *almost everywhere on* $(0,1)$.

Theorem 5.2 *Given $E_0 \in (E_0^*, E_{\min})$. The problem (11) has families of solutions $E_1^+(x,\varepsilon)$, $E_k^\pm(x,\varepsilon)$, $k = 2,3,...,$ defined for sufficiently small ε; these families have the following properties: $\lim\limits_{\varepsilon\to 0+0} E_1^+(x,\varepsilon) = E_2(E_0)$ uniformly for every compact set from $(0,1)$;*

$\lim\limits_{\varepsilon\to 0+0} E_k^\pm(x,\varepsilon) = E_2(E_0)$ *almost everywhere on* $(0,1)$.

The asymptotic behaviour of the eigenfunctions of the problem (11) sharply changes when $E_0 = E_0^*$. In this case the families of solutions have interior transition points.

Definition 5.1 [2]. *Let $E(x,\varepsilon)$ be the famili of solutions for the problem (10), defined for sufficiently small $\varepsilon > 0$. Point $x_0 \in (0,1)$ is called the transition point(interior transition one), if for some sufficiently eni $\delta > 0$ the condition*

$$\lim_{\varepsilon\to 0+0} E(x,\varepsilon) = \begin{cases} E_1(E_0) & \textit{uniformlu for } 0 < x < x_0 - \delta, \\ E_2(E_0) & \textit{uniformly for } x_0 + \delta < x < 1 \end{cases}$$

is fulfilled (or an analogous condition where $E_2(E_0)$ and $E_1(E_0)$ are inserted in place of $E_1(E_0)$ and $E_2(E_0)$).

Theorem 5.3 states that there exists a family of solutions of the problem (11) with an arbitrary large number of transition points of such solutions.

Theorem 5.3 *Given $E_0 = E_0^*$. The problem (11) has families of solutions $E_k^\pm(x,\varepsilon)$, $k = 1,2,...$ defined for sufficiently small ε; moreover every family $E_k^\pm(x,\varepsilon)$ has exactly $k-1$ transition points on the interval $(0,1)$.*

The proves of these theorems follow from some results of R.O'Malley [7]. It appears that the asymptotic behaviour of the solutions of the problem (11) depends essentially on the properties of such differential equations as

$$\ddot{y} + \sqrt{\varepsilon}\dot{y}y - H(y,E_0) = 0, \tag{12}$$

$$\ddot{u} - H(u,E_0) = 0. \tag{13}$$

Proposition 5.1 *Given $E_0 \in (E_{\max}, E_0^*)$. For sufficiently small $\varepsilon > 0$ the differential equation (12) has a solution $y(t,\varepsilon)$ such that $\lim\limits_{t\to+\infty} y(t,\varepsilon) =$*

$\lim\limits_{t\to-\infty} y(t,\varepsilon) = E_1(E_0)$, *and*

$$\lim_{\varepsilon\to 0+0} y(t,\varepsilon) = y_0(t), \quad t \in R,$$

where $y_0(t)$ *is the solution of the equation (13); for this solution*

$$\lim_{t\to+\infty} y_0(t) = \lim_{t\to-\infty} y_0(t) = E_1(E_0).$$

Proof. Since $E_0 \in (E_{\max}, E_0^*)$, then proposition 2.2 gives $\int\limits_{E_1(E_0)}^{E_2(E_0)} H(s,E_0)ds < 0$. Therefore from the proposition 5 (see the theorem from [7]) it can be obtained that the equation(13) has a solution $y_0(t)$ with the properties mentioned above.

Rewrite for convienience the equation (12) as

$$G_1(\ddot{y},\dot{y},y,\varepsilon) = \ddot{y} + \varepsilon\dot{y}y - H(y,E_0) = 0. \tag{14}$$

Substitation $y - y_0 = v$ gives

$$G_1(\ddot{y}_0+\ddot{v},\dot{y}_0+\dot{v},y_0+v,\varepsilon) = \ddot{y}_0+\ddot{v}+\varepsilon(\dot{y}_0+\dot{v})(y_0+v) - H(y_0+v) = 0, \tag{15}$$

where $\lim\limits_{|t|\to\infty} v(t,\varepsilon) = 0$.

Lemma 4.2 [3] states that the left–hand part of (12) defines on operator $\tilde{G}_1(v,\varepsilon)$ from $X\times R^1$ into Y, where $X = H_2\cap C^2$, $Y = H_0 \cap C^0$ with the norms $\|v\|_X = |v|_2 + \left(\sum\limits_{k=0}^{2}\int\limits_{-\infty}^{+\infty} |v^{(k)}(\eta)|^2 d\eta\right)^{1/2}$, $\|v\|_Y = |v|_0 + \left(\int\limits_{-\infty}^{+\infty} v^2(\eta)d\eta\right)^{1/2}$. Let us verify the condition of lemma 3.1 [3]:

(i) $M \equiv \tilde{G}_1$, $m(v,\varepsilon) \equiv v(0)$, $\tilde{G}_1(0,0) = 0$, $m(0,0) = 0$;

(ii) $\Phi = \dot{y}_0 \in X$, $\langle\Phi^*, v\rangle = \int\limits_{-\infty}^{+\infty} \dot{y}_0 v d\eta$,

$R(M_1(0,0)) = \{v \in Y : \langle\Phi^*, v\rangle = 0\}$, ΘΛΞ $M_1(0,0)w = \ddot{w} - H'(y_0)w$;

(iii) $\langle\Phi^*, M_2(0,0;1)\rangle = \int\limits_{-\infty}^{+\infty} \dot{y}_0\dot{y}_0 y_0 d\eta \neq 0$;

(iv) $m_1(0,0;\Phi) = \Phi(0) \neq 0$.

From this lemma follows proposition 4.1. The next proposition is proved by the same way.

Proposition 5.2 *Given* $E_0 \in (E_0^*, E_{\min})$*. For sufficientlu small* $\varepsilon > 0$ *the differential equation (12) has a solution* $y(t,\varepsilon)$ *such that* $\lim\limits_{t\to+\infty} y(t,\varepsilon) = \lim\limits_{t\to-\infty} y(t,\varepsilon) = E_2(E_0)$*, and* $\lim\limits_{\varepsilon\to 0+0} y(t,\varepsilon) = y_0(t)$*,* $t \in R$*, where* $y_0(t)$ *is the solution of the equation (13); for this solution* $\lim\limits_{t\to+\infty} y_0(t) = \lim\limits_{t\to-\infty} y_0(t) = E_2(E_0)$*.*

We have on hands the situation where $E_0 = E_0^*$.

Proposition 5.3 *Given* $E_0 = E_0^*$*. For sufficientlu small* $\varepsilon > 0$ *the diferetial equation (12) has the solution* $y(t,\varepsilon)$ *such that* $\lim\limits_{t\to+\infty} y(t,\varepsilon) = E_2(E_0^*)$*,* $\lim\limits_{t\to-\infty} y(t,\varepsilon) = E_1(E_0^*)$ *and* $\lim\limits_{\varepsilon\to 0+0} y(t,\varepsilon) = y_0(t)$*,* $t \in R$*, where* $y_0(t)$ *is the solution of (13), for which* $\lim\limits_{t\to+\infty} y_0(t) = E_2(E_0^*)$*,* $\lim\limits_{t\to-\infty} y_0(t) = E_1(E_0^*)$*.*

Proof. We shall use the results of P.Fife [3]. The equation $\varepsilon E'' + \varepsilon E'E - H(E, E_0^*) = 0$ can be rewritten as $F(\varepsilon E'', \sqrt{\varepsilon}E', E, \varepsilon) = 0$. Substitution $t = \dfrac{x-c}{\sqrt{\varepsilon}}$, where c is an arbitrary constant gives

$$F(\varepsilon E'', \sqrt{\varepsilon}E', E, \varepsilon) \equiv G(\ddot{y}, \dot{y}, y, \varepsilon) = \ddot{y} + \sqrt{\varepsilon}\dot{y}y - H(y, E_0^*) = 0. \tag{16}$$

The function $y_0(t) = y(t, 0)$ satisfies the equation

$$G(\ddot{y}_0, \dot{y}_0, y_0, 0) = \ddot{y}_0 - H(y, E_0^*) = 0. \tag{17}$$

The properties c) and d) of proposition 2.2 are equvalent to the fast that the equation (17) has the solution $y_0(t)$ satisfying the conditions $y_0(-\infty) = E_1(E_0^*)$, $y_0(+\infty) = E_2(E_0^*)$. It appears that the conditions c) and d) from proposition 2.2 are sufficient for the existence of the solution $y(t,\varepsilon)$ of the equation (16). This solution is defined for sufficiently small $\varepsilon > 0$ and has properties $y(-\infty,\varepsilon) = E_1(E_0^*)$, $y(+\infty,\varepsilon) = E_2(E_0^*)$. This fact can be obtained from the following cpecisl case of theorem 5.1 [3].

Proposition 5.4 ([3]) *The conditions*

1) $H'_E\,(E_i(E_0^*), E_0^*) > 0$*,* $i = 1, 2$*;*

2) $$\int\limits_{E_1(E_0^*)}^{E} H(s, E_0^*)ds \begin{cases} > 0, & \text{for } E \in (E_1(E_0^*), E_2(E_0^*)), \\ = 0, & \text{for } E = E_2(E_0^*) \end{cases}$$

are sufficient for the existence of the family of solutions $y(t,\varepsilon)$ *of the differental equation (16); these solutions are defined for* $t \in R$ *and for sufficientlu small*

$\varepsilon > 0$ *and are uniformly continious on* ε *; for them the properties* $y(-\infty, \varepsilon) = E_1(E_0^*)$, $y(+\infty, \varepsilon) = E_2(E_0^*)$ *are fulfilled.*

Let us return to the proof of 5.1. It is raiher easy to prove that the rest point $(E_0, 0)$ is a stable focus for sufficiently small $\varepsilon > 0$. By C_ε we detnote a closed loop which circles the point $(E_0, 0)$, coming out from the point $(E_1(E_0), 0)$ and returning t_0 it. Then there exists the trajectory $y(t, \varepsilon)$ within the domain which is formed by the loop C_ε; this trajectory has properties: $\omega(y(t, \varepsilon)) = (E_0, 0)$, $\alpha(y(t, \varepsilon)) = C_\varepsilon$, where $\alpha(y)$ and $\omega(y)$ are α–limit set and ω–limit set of $y(t, \varepsilon)$ [8]. Consider an arbitrary solution $E(x, \varepsilon)$ of the problem (11). The function $y(t, \varepsilon)$ is a solution for the boundary value problem

$$\begin{gathered} \ddot{y} + \sqrt{\varepsilon}\dot{y}y - H(y, E_0) = 0, \\ y(0) = E_0, \qquad y\left(-\frac{1}{\sqrt{\varepsilon}}\right) = E_0 \end{gathered} \tag{18}$$

Since $\alpha(y(t, \varepsilon)) = C_\varepsilon \supset (E_1(E_0), 0)$ and the point $(E_1(E_0), 0)$ is a unique saddle point with a separatrix C_ε then theorem 5.1 can be obtained by the reasoning used in the 5–th proposition of R. 'Malley's theorem [7]. Proof of theorem 5.2 completely repeats that of theorem 5.1.

The closed loop C_ε is an α-limit set for any solution of the problem (18). When $E_0 = E_0^*$ C_ε has two saddle points $(E_1(E_0^*), 0)$ and $(E_2(E_0^*), 0)$, this fact and the 4-th proposition of 'Malley's theorem [7] give theorem 5.3.

6. Nonstationary initial-boundary value problem. Existence and uniqueness of solutions for $t > 0$

It is easy to prove that the problem (4) is a stationary problem for the following nonstationary problem [8]:

$$\begin{gathered} D(|E|)^{-1}\frac{\partial E}{\partial t} = \frac{\partial^2 E}{\partial x^2} + \frac{\partial E}{\partial x}E - fH(E, E_0), \\ E(0, t) = E(1, t) = E_0, \\ E(x, 0) = \tilde{E}(x). \end{gathered} \tag{19}$$

The problem (19) is equvalent to the following boundary value problem

$$\begin{gathered} D(|u + E_0|)^{-1}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + E_0\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x}u - fh(u, E_0), \\ u(0, t) = u(1, t) = 0, \\ u(x, 0) = u_0(x), \end{gathered} \tag{20}$$

where $h(u, E_0) = H(u + E_0, E_0)$, $u_0(x) = \tilde{E}(x) - E_0$.

Consider the space $X = L_2(0, 1)$ and the operator $A = -\dfrac{d^2}{dx^2} - -E_0\dfrac{d}{dx}$ with the domain of definition $D(A) = H^2(0, 1) \cap H_0^1(0, 1) \Upsilon D(A^{1/2}) = = X^{1/2} = H_0^1(0, 1)$. It is to prove that for the operator $F\colon H_0^1(0, 1) \to L_2(0, 1)$, which defined by the formyla

$$F(\varphi)(x) = \varphi(x)\varphi'(x) - fh(\varphi(x), E_0), \qquad 0 < x < 1,$$

the conditions of theorems 3.3.3 and 3.3.4 [8] are fulfilled. To this end it is sufficiently to prove that

1) $\|F(\varphi)\|_{L_2} \le c\|\varphi\|^2_{H_0^1}$, i.e. F maps bounded subsets $H_0^1(0, 1)$ into bounded subsets $L_2(0, 1)$;

2) F is Lipschitzian locally.

Using the theorems 3.3.3 and 3.3.4 [8] we can formulate the following proposition.

Proposition 6.1 *A unique solution $u(x, t)$ of Cauchi problem (20) exists on some maximal interval $0 \le t \le \bar{t}$.Moreover this solution exists for any initial condition and either $\bar{t} = +\infty$ or $\|u(x, t)\|_{H_0^1} \to +\infty$ for $t \to \bar{t}$.*

To prove the following proposition we need the theorem 3.5.2. [8] on a smoothing differential operator.

Proposition 6.2 *The solution $u(x, t)$ of the nonstationary problem (20) is a classical solution.*

Proof. $u(t; u_0) \in D(A)$ for $t > 0$. The function $t \mapsto \dfrac{du}{dt} \in \in H_0^1(0, 1)$ is Gölderian locally (see the theorem 3.5.2 [8]). There fore the function

$$(x, t) \mapsto u(x, t; u_0), \quad \frac{\partial u}{\partial t}(x, t; u_0)$$

are continions for $t_0 < t < \bar{t}$ and $x \in [0, 1]$. Since $u \in D(A)$, then $u' \in W_2^1(0, 1) \subset C(0, 1)$. There exists some $\delta > 0$ that $F(u) \in C^\delta(0, 1)$ $u(\cdot, t) \in C^{2+\delta}(0, 1)$. Thus, when $t > 0$ the function $(x, t) \mapsto u(x, t; u_0)$ is continiously differentiable by t and twice continiouslu differentiable by x; therefore it is a classical solution of the (20).

The main item of this section is the following proposition which based on the concept of the dynamical system for parabolic equations [8].

Proposition 6.3 *The nonstationary problem (20) defines the dynamical* *system in the set* $C = \left\{ u \in H_0^1(0,1) \middle| \sigma_1(E_0) \le u(x) \le \sigma_2(E_0) \text{ almost every-} \right.$ *where on* $\left.[0,1]\right\}$, *where* $\sigma_i(E_0) = E_i(E_0) - E_0$, $i = 1, 2$.

Proof. First consider the fact that the solution $u(x,t)$ of the problem (20) with the initial condition $u_0 \in C$ can not leave the set C on the interval of its existence. For this we use a version of the maximum principle. Let t_1 be a minimal value $t_1 \in (0, \bar{t})$, so that the solution $u(x, t_1)$ of the problem (20) has a local maximum. $\sigma = u(x_1, t_1)$, where $x_1 \in (0,1)$ and $\sigma > \sigma_2(E_0) > 0$, i.e. $\frac{\partial u}{\partial x}(x_1, t_1) = 0$, $\frac{\partial^2 u}{\partial x^2}(x_1, t_1) \le 0$. Then from the differential equation (20) we obtain that since $(-h(\sigma, E_0)) < 0$ then $\dot{u}(x_1, t_1) < 0$.

Furthermore, the solution $u(x,t)$ of the problem (20) can not have a local minimum, which value is less than $\sigma_1(E_0) < 0$ (a proof is quite similar). Prove that the solution of (20) exists for any $t \ge 0$. Let it be not so. Then proposition 6.1 gives that $\int\limits_0^1 u'^2(x,t)dx$ is unbounded for $t \to \bar{t}$. Multiplying the equation (20) by u and integrating its parts on $[0,1]$ gives

$$\int\limits_0^1 \frac{\dot{u}u}{D(|u + E_0|)} dx = -\int\limits_0^1 u'^2 dx - f \int\limits_0^1 h(u, E_0) u dx.$$

But $\left(-f \int\limits_0^1 h(u, E_0) u dx \right)$ is uniformly bounded for t; then $\int\limits_0^1 \frac{\dot{u}u}{D(|u + E_0|)} dx \to -\infty$ for $t \to \bar{t}$. But the latter statement is impossible because Gronuollo inequality $\dot{\varphi} \le -c_1 \varphi + c_2$ $(c_i > 0,\ i = 1, 2)$ is valid for function $\varphi(t) = \int\limits_0^1 g(u(x,t))dx$, where $g(u) = \int\limits_0^u s D^{-1}(|s + E_0|) ds$.

7. Stability and instability of eigenfunctions

This section deals with stability of solutions for the stationary problem (4) by the linear approximetion. For this aim we need two theorems (5.1.1 and 5.1.3) from [8], which give sufficient conditions of stability and instability of such solutions. Let $E_k^\nu(x,\varepsilon)$ be the solutions of the problem (11) from section 4 defined for sufficienly small ε, $\nu = +,-$; $k = 1, 2, ...$ Two following theorems form the main subject of this section.

Theorem 7.1

1) If $E_{\max} < E_0 < E_0^*$, then the solution $E_1^-(x,\varepsilon)$ of the problem (11) is stable for sufficiently small ε;

2) if $E_0^* < E_0 < E_{\min}$, then the solution $E_1^+(x,\varepsilon)$ of the problem (11) is stable for sufficiently small ε;

3) if $E_0 = E_0^*$, then both solutions $E_1^\pm(x,\varepsilon)$ of the problem (11) are stable for sufficiently small ε;

Theorem 7.2 *The solution $E_k^\pm(x,\varepsilon)$ are unstable for $E_k^\pm(x,\varepsilon)$, for every $E_0 \in (E_{\max}, E_{\min})$ and for each $k = 2, 3, ...$*

Remark. Theorem 7.1 states thates that the solutions are stable only for sufficiently small $\varepsilon > 0$. Apparently this fact is assignable; it is quite possible that the solutions $E_1^\pm(x,\varepsilon)$ can be unstable when ε is not small.

Proves of Theorems 6.1 and 6.2 must be preceeded by rather spacious preambula. First discuss the case where $E_{\max} < E_0 < E_0^*$. For such E_0

$$\int\limits_{E_1(E_0)}^{E_2(E_0)} H(s, E_0)ds < 0$$

and the problem

$$\begin{gathered} \hat{z}_0'' = H(E_1(E_0) + \hat{z}_0, E_0), \\ \hat{z}_0(0) = E_0 - E_1(E_0), \quad \hat{z}_0(+\infty) = 0 \end{gathered} \tag{21}$$

has a unique solution $\hat{z}_0(t)$ which is a strong monotonous function [7]. Let $\zeta(y)$ is a cut-off function from C^∞-class and the conditions $0 \le \zeta \le 1$, $\zeta \equiv 1$ for $0 \le y \le 1/4$, $\zeta \equiv 0$ for $y \ge \dfrac{1}{2}$, are fulfilled for it. Let $z_0(x,\varepsilon) = \hat{z}_0\left(\dfrac{x}{\sqrt{\varepsilon}}\right)\zeta(x)$, $z_1(x,\varepsilon) = \hat{z}_0\left(\dfrac{1-x}{\sqrt{\varepsilon}}\right)\zeta(1-x)$, $0 \le x \le 1$. Consider the function

$$U_0(x,\varepsilon) = E_1(E_0) + z_0(x,\varepsilon) + z_1(x,\varepsilon).$$

This function (see theorem 5.1) is a first approximation on ε for $E_1^-(x,\varepsilon)$. When $E_0^* < E_0 < E_{\max}$ the function $U_0(x,\varepsilon)$ can be formed quite analogically($E_2(E_0)$ stands in the place of $E_1(E_0)$) and it is a first approximation on ε for $E_1^+(x,\varepsilon)$.

We have on hands the case $E_0 = E_0^*$. So as $\int\limits_{E_1(E_0^*)}^{E_2(E_0^*)} H(s, E_0^*)ds = 0$ only for this condition each of the problems

$$\hat{z}_0'' = H(E_i(E_0^*) + \hat{z}_0, E_0^*),$$
$$\hat{z}_0(0) = E_0^* - E_i(E_0^*), \quad \hat{z}_0(+\infty) = 0, \quad i = 1,2$$

has a unique solution $\hat{z}_0^{(i)}(t)$ $(i = 1,2)$ which is a stroug monotonous [7]. Let $z_0^{(i)}(x,\varepsilon) = \hat{z}_0^{(i)}\left(\dfrac{x}{\sqrt{\varepsilon}}\right)\zeta(x)$ and $\hat{z}_1^{(i)}(t)$ is the unique strong monotonous solution of the problem

$$\hat{z}_1'' = H(E_i(E_0^*) + \hat{z}_1, E_0^*),$$
$$\hat{z}_1(0) = E_0^* - E_i(E_0^*), \quad \hat{z}_1(+\infty) = 0, \quad i = 1,2$$

Let $z_1^{(i)}(x,\varepsilon) = \hat{z}_1^{(i)}\left(\dfrac{1-x}{\sqrt{\varepsilon}}\right)\zeta(1-x)$, $0 \le x \le 1$. Consider the function $U_0^{(i)}(x,\varepsilon) = E_i(E_0^*) + z_0^{(i)}(x,\varepsilon) + z_1^{(i)}(x,\varepsilon)$, $i = 1,2$. Theorem4.1 states that they are the first approximation on ε for the solutions $E_1^{\pm}(x,\varepsilon)$ of the problem (11). For $\varepsilon > 0$ and $u \in C_0^{(2)}$ the norm

$$|u|_2^{(\varepsilon)} = |u|_0 + \sqrt{\varepsilon}|u'|_0 + \varepsilon|u''|_0$$

can be introduced, where $|\cdot|_0$ is a norm in $C^{(0)}$; let the corresponding Banach space be denoted by $C_{0,\varepsilon}^{(2)}$. The linear operator mapping from $C_{0,\varepsilon}^{(2)}$ into $C^{(0)}$ can be constructed:

$$L_\varepsilon u = \varepsilon u'' + \varepsilon U_0' u + \varepsilon U_0 u' - H'(U_0, E_0)u,$$

(the function $U_0(x,\varepsilon)$ is constructed according to the value of E_0 by means which where used above). The linear operators

$$L_\varepsilon^{(i)} u = \varepsilon u'' + \varepsilon {U_0^{(i)}}' u + \varepsilon U_0^{(i)} u' - H'(U_0^{(i)}, E_0^*)u, \quad (i = 1,2) are formed similarly.$$

Lemma 71.. *Operators $L_\varepsilon^{(i)}$, $i = 1,2$ and L_ε have inverse ones, which are uniformly bounded on ε for sufficiently small $\varepsilon > 0$.*

Proof. For the proof it is sufficient to show that there exists a constant c independent of ε, shuch that for any continuous function F with $|F|_0 \leq 1$ and for any sufficiently small $\varepsilon > 0$ there exists solution u_ε for

$$\begin{gathered} L_\varepsilon u_\varepsilon = F(x), \quad 0 \leq x \leq 1, \\ u_\varepsilon(0) = u_\varepsilon(1) = 0, \end{gathered} \tag{22}$$

satisfying $|u_\varepsilon|_2^{(\varepsilon)} \leq c$. This can be done by constructing supersolutions $\overline{u}_\varepsilon$ and subsolutions $\underline{u}_\varepsilon$. For this supersolution the conditions $L_\varepsilon \overline{u}_\varepsilon \leq F$, $\overline{u}_\varepsilon \geq 0$, $\overline{u}_\varepsilon(1) \geq 0$ must be valid by definition. The opposite inequalities must be valid for the subsolution. If the positive supersolution can be constructed, then we merely take $|u_\varepsilon|_0 \leq |\overline{u}_\varepsilon|_0$. By a theorem of Nagumo [9], there exists an exact solution (22) with $|u_\varepsilon|_2^{(\varepsilon)} \leq \text{const}|\overline{u}_\varepsilon|_0$. This inequality and equation (22) together with interpolation inequality relating $|u''|_0$, $|u'|_0$ and $|u|_0$ given that $|u_\varepsilon|_2 \leq const|\overline{u}_\varepsilon|_0$. The positive supersolution $\overline{u}_\varepsilon$ with $|\overline{u}_\varepsilon|_0$ uniformly bounded on ε can be constructed like that in the Lemma 2.1 [2].

Proof of theorem 7.1. For definiteness we shall presume that $E_{\max} < E_0 < E_0^*$ and prove stability of solution $E_1^-(x, \varepsilon) = E_1^-(x, f^{-1}) = \tilde{E}_1^-(x, f)$. Consider the followig boudary value problem

$$\begin{gathered} v_0'' + \tilde{E}_1^{-\prime} v_0 + \tilde{E}_1^- v_0' - fH'(\tilde{E}_1^-, E_0)v_0 + \alpha_0 v_0 = 0, \\ v_0(0) = v_0(1) = 0, \end{gathered}$$

where v_0 is the first positive eigenfunction of the linear operator $L_f u = u'' + \tilde{E}_1^{-\prime} u + \tilde{E}_1^- u' - fH'(\tilde{E}_1^-, E_0)u$. It is easy to show that the spectrum of the L_f operator is real. Theorem 5.1.1 [8] states that, if $\alpha_0 > 0$ then solution $\tilde{E}_1^-(x, f)$ is stable. Consider the problem

$$\psi'' + \tilde{E}_1^{-\prime}\psi + \tilde{E}_1^-\psi' - fH'(\tilde{E}_1^-, E_0)\psi = 0. \tag{23}$$

From 7.1 it followes that for sufficiently large f the problem(23) has only trivial solution. Theorem of Nagumo [9] states that $\alpha_0 > 0$ in fast, suppose that $\alpha_0 < 0$ than we have $L_f v_0 \geq L_f \psi = 0$. Hence $v_0 \leq \psi(x) \equiv 0$ which is impossible. Cases when $E_0^* < E_0 < E_{\min}$ and $E_0 = E_0^*$ can be prove similarly.

Proof of the theorem 7.2. We use here the following proposition.

Proposition 7.1 ([8]) it Let the function $\varphi(x)$, $\psi(x) \in C^2$ satisfy the conditions $\varphi(0) = \psi(0) = 0$, $\varphi'(0) = \psi'(0) = 1$, $\varphi'' + b(x)\varphi' + +a(x)\varphi > \psi'' + b(x)\psi' + a(x)\psi$ for $0 < x < x_1$ and $\psi(x) > 0$ for $0 < x < x_1$. Then it follows $\varphi(x) > \psi(x)$ for $0 < x \leq x_1$.

Let $\varphi_k^{\pm}(x,f) = E_k^{\pm\prime}(x,f^{-1}) = \tilde{E}_k^{\pm\prime}(x,f)$, $k = 2,3,...$ If $\psi(x)$ is the solution of this problem

$$\psi'' + \tilde{E}_k^{\nu\prime}\psi + \tilde{E}_k^{\nu}\psi' - fH'(\tilde{E}_k^{\nu}, E_0)\psi = 0,$$
$$\psi(0) = 0, \quad \psi'(0) = 1,$$

$\nu = +, -$, $k \geq 2$ then

$$\psi(x)\varphi_k^{\nu\prime}(x) - \psi'(x)\varphi_k^{\nu}(x) = ce^{-\int\limits_0^x \tilde{E}_k^{\nu}(s)ds},$$

where $c = \text{const}$. So far as $\varphi_k^{\nu}(0) > 0$, then $c < 0$. Hence $x \in [0,1]$ $\psi(x)\varphi_k^{\nu\prime}(x) - \psi'(x)\varphi_k^{\nu}(x) < 0$. Let x_0 be the minimum point of the function $\tilde{E}_k^{\nu}(x)$. Consequently $\psi(x_0)\varphi_k^{\nu\prime}(x_0) < 0$ because $\varphi_k^{\nu}(x_0) = 0$, $\varphi_k^{\nu\prime}(x_0) > 0$. Therefore, $\psi(x_0) < 0$ and $\psi(x)$ has negative values on $[0,1]$. Consider the following problem

$$v_0'' + \tilde{E}_k^{\nu\prime}v_0 + \tilde{E}_k^{\nu}v_0' - fH(\tilde{E}_k^{\nu}, E_0)v_0 + \alpha_0 v_0 = 0,$$
$$v_0(0) = v_0(1) = 0,$$

where v_0 is the first positive eigenfunction of the linear operator $L_f u = u'' + \tilde{E}_k^{\nu\prime}u + \tilde{E}_k^{\nu}u' - fH'(\tilde{E}_k^{\nu}, E_0)u$. Suppose that $v_0'(0) = 1$. Theorem 5.1.3 [8] states that if $\alpha_0 < 0$, then the solution $\tilde{E}_k^{\nu}(x,f)$ is unstabile. In fact, let $\alpha_0 > 0$, then proposition 7.1 it follows that $v_0(x) < \psi(x)$ for every $x \in [0,1]$ because $L_f v_0 < L_f \psi$. This is impossible.

8. A parabolic traveling wave. Stability of the traveling wave

Consider the parabolic equation of the problem (19) for $E_0 = E_0^*$

$$D(|E|)^{-1}\frac{\partial E}{\partial t} = \frac{\partial^2 E}{\partial x^2} + \frac{\partial E}{\partial x}E - fH(E, E_0^*), \quad x \in R, \quad t > 0. \tag{24}$$

It is easy to show that, if $\varphi(s)$ is solution of the equation

$$\varphi'' + \left(\varphi - \frac{V}{D(|\varphi|)}\right)\varphi' = fH(\varphi, E_0^*), \quad s \in R, \tag{25}$$

than $E(x,t) = \varphi(x + Vt)$ is a traveling wave ($V = const$). We shall prove that for sufficiently large f, there exists a solution of (25) such that $\varphi(s) \to E_1(E_0^*)$

for $s \to -\infty$, $\varphi(s) \to E_2(E_0^*)$ for $s \to +\infty$. Divide f the equation (25) , let $\varepsilon = f^{-1}$ and suppose $\tau = \frac{s}{\sqrt{\varepsilon}}$. Then we have

$$\ddot{y} + \sqrt{\varepsilon}\left(y - \frac{V}{D(|y|)}\right)\dot{y} = H(y, E_0^*). \tag{26}$$

Theorem 4.1 [3] states that for sufficiently small ε the equition (26) has a solution $y(\tau, \varepsilon)$ such that $y(\tau, \varepsilon) \to E_1(E_0^*)$ for $\tau \to -\infty$, $y(\tau, \varepsilon) \to E_2(E_0^*)$ for $\tau \to +\infty$. The folloving interesting result [8] states the stability of parabolic traveling wave. Suppose that there exists a solution $\varphi(x)$

$$\varphi''(x) + f\left(\varphi(x), \varphi'(x)\right) = 0, \quad x \in R,$$

shuch that $\varphi(x) \to \alpha$ for $x \to -\infty$, $\varphi(x) \to \beta$ for $x \to +\infty$ $f(u,p) \in C^1$ and $f(\alpha, 0) = f(\beta, 0) = 0$. The linearized problem of φ is

$$-Lv = v'' + a(x)v' + b(x)v,$$

where $a(x) = \frac{\partial f}{\partial p}\left(\varphi(x), \varphi'(x)\right)$, $b(x) = \frac{\partial f}{\partial u}\left(\varphi(x), \varphi'(x)\right)$. Let $a_\pm$, $b_\pm$ be the limits of these functions for $x \to \pm\infty$. Denote by $\sigma_e(L)$ the essential spectrum of operator L [8].

Proposition 8.1 ([8]) *The essential spectrum $\sigma_e(L)$ lies in right semiplane if and only if $b_+ < 0$ and $b_- < 0$, i.e. where the solution $\varphi(x)$ connects two saddle points.*

Proposition 8.2 ([8]) *If the solution $\varphi(x)$ connects two saddle points, then the solution $\varphi(x)$ is stabile.*

Since $E_1(E_0^*)$, $E_2(E_0^*)$ are saddle points, then proposition 8.1 and proposition 8.2 state that for sufficiently small ε the solution $y(\tau, \varepsilon)$ of the equation (26) connects two saddle points. The exercise 6 in § 5.1 [8] gives the result that the solution $E(x,t)$ expotentially approximates φ by norm in φ ΔΩff Ξfiff ΩΔ $W_p^1(R)$, $p \geq 1$. It means that for every solution $E(x,t)$, such that the norm $\|E(\cdot, 0) - \varphi\|$ in $W_p^1(R)$, $p \geq 1$ is sufficiently small, there exists a real c for which $\|E(\cdot, t) - \varphi(\cdot + c + Vt)\| = O\left(e^{-\beta t}\right)$, $t > 0$, $\beta > 0$. Finally, we obtain the stability of the parabolic traveling wave for sufficiently large f and for arbitrary velocity V.

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