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Integrodifferential Equations

Existence of Solutions of Quasilinear Mixed Volterra-Fredholm Integrodifferential Equations with Nonlocal Conditions

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Abstract: In this paper, we establish the existence and uniqueness of mild and classical solutions of quasilinear mixed Volterra-Fredholm integrodifferential equation with nonlocal condition in Banach space. The results are established by using the semigroup theory and Banach fixed point theorem.

Keywords: Mild and Classical solutions, Banach fixed point theorem, Nonlocal condition, Mixed Volterra-Fredholm, Integrodifferential equation.

2000 Mathematics Subject Classification: 47H10, 34G20, 47D03, 45N05.

1 Introduction

Byszewski [6] has established the existence and uniqueness of mild, strong and classical solutions of the following nonlocal Cauchy problem

$$\frac{du(t)}{dt} + Au(t) = f\big(t, u(t)\big), \quad t \in (0, a], \tag{1}$$

$$u(0) + g\left(t_1, t_2, \ldots., t_p, u(t_1), u(t_2), \ldots., u(t_p)\right) = u_{0,} \tag{2}$$

where $-A$ is the infinitesimal generator of a C_0-semigroup $T(t), t \geq 0$, in a Banach space X, $0 < t_1 < \cdots, < t_p \leq a, a > 0, u_0 \in X$ and $f: [0, a] \times X \to X, g: [0, a]^p \times X^p \to X$ are given functions. Subsequently many authors extended the work to various kinds of nonlinear evolution equations [3, 4, 7, 8].

Abstract quasilinear integrodifferential equations arise in many areas of science such as population dynamics, mathematical physics, heat conduction theory of materials with memory etc. For this reason, such types of equations have received much attention in recent years. The literature related to quasilinear differential and integrodifferential equations is very extensive.

T. Kato [12] has proved two general theorems on the nonhomogeneous quasilinear evolution equations of the type

$$u'(t) + A\big(t, u(t)\big)u(t) = f\big(t, u(t)\big), \quad 0 < t \le T, \quad u(0) = u_0, \tag{3}$$

one on the existence and uniqueness, and the other on the continuous dependence of a solution on the initial data.

Crandall and Souganidis [9] have used a different method to prove the existence, uniqueness and continuous dependence of a continuously differentiable solution to the quasilinear evolution equation

$$u'(t) + A\big(u(t)\big)u(t) = 0, \quad 0 < t \le T, \quad u(0) = u_0,$$

under similar assumptions considered by T. Kato [12].

Pazy [15] considered the following quasilinear equation of the form

$$u'(t) + A(t, u)u(t) = 0, \quad 0 < t \le T, \quad u(0) = u_0,$$

and discussed the mild and classical solutions by using the fixed point argument. Bahuguna [1, 2], Oka [13], and Oka and Tanaka [14] discussed the existence of solutions of quasilinear integrodifferential equations in Banach spaces. Recently Balachandran and Samuel [5] studied the existence of solutions for quasilinear delay integrodifferential equations with nonlocal conditions by using C_0-semigroup and the Banach fixed point theorem. Shengli Xie [16] studied the existence of solutions for nonlinear mixed type integrodifferential functional evolution equations with nonlocal conditions using Monch fixed point theorem. Dhakne and Ku*cche [10] prove the existence, uniqueness and continuous dependence of mild solution of a nonlinear mixed Volterra-Fredholm functional integrodifferential equation with nonlocal condition by using semigroup theory and Banach fixed point theorem. An equation of this type occurs in a nonlinear conservation law with memory

$$u(t,x) + \psi\big(u(t,x)\big)_x = \int_0^t b(t-s)\psi\left(u(t,x)\right)_x ds + f(t,x), \quad t \in [0,a], \ x \in R \tag{4}$$

$$u(0,x) = \emptyset(x), \qquad x \in R \tag{5}$$

It is clear that if nonlocal condition (2) is introduced to (4), and then it will also have better effect than the classical condition $u(0,x) = \emptyset(x)$. Therefore, we would like to extend the results for (1) – (2) to a class of integrodifferential equations in Banach spaces.

In this paper we study the following quasilinear mixed Volterra-Fredholm integrodifferential equation with nonlocal condition of the form

$$u'(t) + A(t,u)u(t) = f\left(t, u(t), \int_0^t k\big(t,s,u(s)\big)ds, \int_0^a h\big(t,s,u(s)\big)ds\right), \quad t \in [0,a] \tag{6}$$

$$u(0) + g(u) = u_0 \tag{7}$$

where $A(t,u)$ is the infinitesimal generator of a C_0-semigroup in a Banach space $X, u_0 \in X, f: J \times X \times X \times X \to X, k, h: \Delta \times X \to X$ and $g: C(J:X) \to X$ are given functions. Here $J = [0,a]$ and $\Delta = \{(t,s): 0 \le s \le t \le a\}$. The results obtained in this paper are generalization of the results given by Pazy [15], Kato [11] and Bahuguna [2].

2 Preliminaries

Let X and Y be two Banach space such that the embedding $Y \hookrightarrow X$ is dense and continuous. The norm in any Banach space Z is denoted by $\|.\|$ or $\|.\|_Z$. The space of all bounded linear operators from a

Banach space X to a Banach space Y is denoted by $B(X,Y)$ and $B(X,X)$ is written as $B(X)$. we remember some definitions and known facts from Pazy [15].

Definition 2.1: Let S be a linear operator in X and let Y be a subspace of X. The operator $\tilde{S}$ defined by $D(\tilde{S}) = \{x \in D(S) \cap Y : Sx \in Y\}$ and $\tilde{S}x = Sx$ for $x \in D(\tilde{S})$ is called the part of S in Y.

Definition 2.2: Let B be a subset of X and for every $0 \le t \le a$ and $b \in B$, let $A(t,b)$ be the infinitesimal generator of a C_0-semigroup $S_{t,b}(s), s \ge 0$, on X. The family of operators $\{A(t,b)\}, (t,b) \in J \times B$, is stable if there are constants $M \ge 1$ and ω such that

$$\rho\big(A(t,b)\big) \supset \,]\omega, \infty[\qquad \text{for } (t,b) \in J \times B,$$

and

$$\left\| \prod_{j=1}^{k} R\left(\lambda : A\big(t_j, b_j\big)\right) \right\| \le M(\lambda - \omega)^{-k}$$

for $\lambda > \omega$ and every finite sequence $0 \le t_1 \le t_2 \dots \dots \le t_k \le a, b_j \in B, 1 \le j \le k$.

Definition 2.3: Let $S_{t,b}(s), s \ge 0$ be the C_0-semigroup generated by $\{A(t,b)\}, (t,b) \in J \times B$. A subspace Y of X is called $A(t,b)$-admissible if Y is invariant subspace of $S_{t,b}(s)$, and the restriction of $S_{t,b}(s)$ to Y is a C_0-semigroup in Y.

For more details of the above mentioned notions, one may refer to the chapters 5 and 6 in Pazy [15]. On the family of operators $\{A(t,b) : (t,b) \in J \times B\}$, we make the same assumptions $(\widetilde{H}_1) - (\widetilde{H}_4)$ considered in 6.6.4 in Pazy [15] for the homogenous quasilinear evolution equation, as restated below.

(H_1) The family $\{A(t,b) : (t,b) \in J \times B\}$ is stable.

(H_2) Y is $A(t,b)$-admissible for $(t,b) \in J \times B$ and the family $\{\tilde{A}(t,b) : (t,b) \in J \times B\}$ of the parts of $A(t,b)$ in Y is stable in Y.

(H_3) For $(t,b) \in J \times B, D\big(A(t,b)\big) \supset Y$, $A(t,b)$ is a bounded linear operator from Y to X and the map $t \mapsto A(t,b)$ is continuous in the $B(Y,X)$ norm $\|.\|_{Y \to X}$ for every $b \in B$.

(H_4) There is a constant L such that

$$\|A(t,b_1) - A(t,b_2)\|_{Y \to X} \le L\|b_1 - b_2\|_X$$

for every $b_1, b_2 \in B$ and $0 \le t \le a$.

A two parameter family of bounded linear operators $U(t,s), 0 \le s \le t \le a$, on X is called an evolution system if the following two conditions are satisfied:

(i) $U(s,s) = I$ and $U(t,r)U(r,s) = U(t,s)$ for $0 \le s \le r \le t \le a$.

(ii) The map $(t,s) \longmapsto U(t,s)$ is strongly continuous for $0 \le s \le t \le a$.

If $u \in C(J,X)$ has values in B and the family $\{A(t,b) : (t,b) \in J \times B\}$ of the operators satisfies the assumptions $(H_1) - (H_4)$ then there exists a unique evolution system $U(t,s;u)$ in X satisfying

(i) $$\|U(t,s;u)\| \le Me^{\omega(t-s)} \tag{8}$$

for $0 \le s \le t \le a$, where M and ω are stability constants;

(ii) $$\frac{\partial^+}{\partial t} U(t,s;u)w\Big|_{t=s} = A\big(s,u(s)\big)w \tag{9}$$

for $w \in Y$, and $0 \le s \le t \le a$;

(iii) $$\frac{\partial}{\partial s} U(t,s;u)w = -U(t,s;u*)A\big(s,u(s)\big)w \tag{10}$$

for $w \in Y$, and $0 \leq s \leq t \leq a$.

Further, there exists a constant $C_1 > 0$ such that for every $u, v \in C(J, X)$ with values in B and for every $w \in Y$, we have

$$\|U(t,s;u)w - U(t,s;v)w\| \leq C_1 \|w\|_Y \int_s^t \|u(\tau) - v(\tau)\| d\tau. \tag{11}$$

For the details of the above mentioned results, one may refer to Theorem 6.4.3 and Lemma 6.4.4 in Pazy [15].

We further assume that

(H_5) For every $u \in C(J, X)$ satisfying $u(t) \in B$ for $0 \leq t \leq a$, we have

$$U(t,s;u)Y \subset Y, \quad 0 \leq s \leq t \leq a$$

and $U(t,s;u)$ is strongly continuous in Y for $0 \leq s \leq t \leq a$.

(H_6) Y is reflexive.

(H_7) For every $(t, b_1, b_2, b_3) \in J \times B \times B \times B, f(t, b_1, b_2, b_3) \in Y$.

(H_8) $g: C(J, B) \to X$ is Lipschitz continuous in X and bounded in Y, that is, there exist constants $G > 0$ and $G_1 > 0$ such that

$$\|g(u)\|_Y \leq G,$$
$$\|g(u) - g(v)\|_X \leq G_1 \max_{t \in J} \|u(t) - v(t)\|_X.$$

For the conditions $(H_9), (H_{10})$ and (H_{11}) let Z be taken as both X and Y.

(H_9) $f: J \times Z \times Z \times Z \to Z$ is continuous and there exist constants $F_1 > 0$ and $F_2 > 0$ such that

$$\|f(t, u_1, v_1, w_1) - f(t, u_2, v_2, w_2)\|_Z \leq F_1(\|u_1 - u_2\|_Z + \|v_1 - v_2\|_Z + \|w_1 - w_2\|_Z),$$
$$F_2 = \max_{t \in J} \|f(t, 0,0,0)\|_Z.$$

(H_{10}) $k: \Delta \times Z \to Z$ is continuous and there exist constants $K_1 > 0$ and $K_2 > 0$ such that

$$\|k(t,s,u_1) - k(t,s,u_2)\|_Z \leq K_1 \|u_1 - u_2\|_Z,$$
$$K_2 = \max\{\|k(t,s,0)\|_Z : (t,s) \in \Delta\}.$$

(H_{11}) $h: \Delta \times Z \to Z$ is continuous and there exist constants $H_1 > 0$ and $H_2 > 0$ such that

$$\|h(t,s,u_1) - h(t,s,u_2)\|_Z \leq H_1 \|u_1 - u_2\|_Z,$$
$$H_2 = \max\{\|h(t,s,0)\|_Z : (t,s) \in \Delta\}.$$

Let us take $\qquad M_0 = \max\{\|U(t,s;u)\|_{B(Z)}, 0 \leq s \leq t \leq a, u \in B\}.$

(H_{12})

$M_0(\|u_0\|_Y + G) + M_0 F_1 ra + M_0 F_1 K_1 ra^2 + M_0 F_1 K_2 a^2 + M_0 F_1 H_1 ra^2 + M_0 F_1 H_2 a^2 + M_0 F_2 a \leq r$

and

$$q = C_1 a \|u_0\|_Y + GC_1 a + M_0 G_1 + C_1 a(F_1 r + F_1 K_1 ar + F_1 K_2 a + F_1 H_1 ar + F_1 H_2 a + F_2)$$
$$+ M_0 F_1 a + M_0 F_1 K_1 a^2 + M_0 F_1 H_1 a^2 < 1.$$

By a mild solution of (6) – (7) we mean a function $u \in C(J, X)$ with values in B and $u_0 \in X$ satisfying the integral equation

$$u(t) = U(t,0;u)u_0 - U(t,0;u)g(u) + \int_0^t U(t,s;u) f\left(s, u(s), \int_0^s k\big(s,\tau,u(\tau)\big)\,d\tau, \int_0^a h\big(s,\tau,u(\tau)\big)\,d\tau\right) ds. \qquad (12)$$

By the classical solution of (6) – (7) we mean a function $u \in C(J,X)$ such that $u(t) \in D\left(A\big(t,u(t)\big)\right)$ for $t \in (0,a], u \in C^1\big((0,a],X\big)$ and satisfies (6) – (7) on J.

3 Main Result

Theorem 3.1: Let $u_0 \in Y$ and let $B = \{u \in X : \|u\|_Y \le r\}, r > 0$. If the assumptions $(H_1) - (H_{12})$ are satisfied, then the quasilinear problem (6) – (7) has a unique classical solution $u \in C([0,a]:Y) \cap C^1\big((0,a]:X\big)$.

Proof: Let S be a nonempty closed subset of $C(J,X)$ defined by

$$S = \{u : u \in C(J,X), \|u(t)\|_Y \le r \; for \; 0 \le t \le a\}.$$

Consider a mapping F on S defined by

$$(Fu)(t) = U(t,0;u)u_0 - U(t,0;u)g(u) + \int_0^t U(t,s;u) f\left(s, u(s), \int_0^s k\big(s,\tau,u(\tau)\big)\,d\tau, \int_0^a h\big(s,\tau,u(\tau)\big)\,d\tau\right) ds.$$

We claim that F maps S into S. For $u \in S$, we have

$$\|Fu(t)\|_Y = \left\| U(t,0;u)u_0 - U(t,0;u)g(u) + \int_0^t U(t,s;u) f\left(s, u(s), \int_0^s k\big(s,\tau,u(\tau)\big)\,d\tau, \int_0^a h\big(s,\tau,u(\tau)\big)\,d\tau\right) ds \right\|$$

$$\le \|U(t,0;u)u_0\| + \|U(t,0;u)g(u)\| + \int_0^t \left\| U(t,s;u) f\left(s, u(s), \int_0^s k\big(s,\tau,u(\tau)\big)\,d\tau, \int_0^a h\big(s,\tau,u(\tau)\big)\,d\tau\right) \right\| ds$$

$$\le \|U(t,0;u)u_0\| + \|U(t,0;u)g(u)\| + \int_0^t \left\| U(t,s;u) \left[f\left(s, u(s), \int_0^s k\big(s,\tau,u(\tau)\big)\,d\tau, \int_0^a h\big(s,\tau,u(\tau)\big)\,d\tau\right) - f(s,0,0,0) + f(s,0,0,0) \right] \right\| ds$$

$$\le M_0\|u_0\|_Y + M_0 G + M_0 \int_0^t \left[F_1\left(\|u(s) - 0\| + \left\| \int_0^s k\big(s,\tau,u(\tau)\big) d\tau - 0 \right\| + \left\| \int_0^a h\big(s,\tau,u(\tau)\big) d\tau - 0 \right\| \right) + \|f(s,0,0,0)\| \right] ds$$

$$\leq M_0\|u_0\|_Y + M_0G + M_0\int_0^t \left[F_1 r + F_1\int_0^s \left\|k\big(s,\tau,u(\tau)\big) - k(s,\tau,0) + k(s,\tau,0)\right\| d\tau\right.$$

$$\left. +F_1\int_0^a \left\|h\big(s,\tau,u(\tau)\big) - h(s,\tau,0) + h(s,\tau,0)\right\| d\tau + F_2\right] ds$$

$$\leq M_0\|u_0\|_Y + M_0G$$

$$+ M_0\int_0^t [F_1 r + F_1 a(K_1 r + K_2) + F_1 a(H_1 r + H_2) + F_2] ds$$

$$\leq M_0\|u_0\|_Y + M_0G + M_0F_1 ra + M_0F_1K_1ra^2 + M_0F_1K_2a^2 + M_0F_1H_1ra^2 + M_0F_1H_2a^2 + M_0F_2a \leq r.$$

Therefore F maps S into itself. Moreover, if $u, v \in S$, then

$$\|Fu(t) - Fv(t)\| \leq \|U(t,0;u)u_0 - U(t,0;v)u_0\| + \|U(t,0;u)g(u) - U(t,0;v)g(v)\|$$

$$+\int_0^t \left\| U(t,s;u)f\left(s,u(s),\int_0^s k\big(s,\tau,u(\tau)\big)\,d\tau,\int_0^a h\big(s,\tau,u(\tau)\big)\,d\tau\right)\right.$$

$$\left. - U(t,s;v)f\left(s,v(s),\int_0^s k\big(s,\tau,v(\tau)\big)\,d\tau,\int_0^a h\big(s,\tau,v(\tau)\big)\,d\tau\right)\right\| ds$$

$$\leq \|U(t,0;u)u_0 - U(t,0;v)u_0\| + \|U(t,0;u)g(u) - U(t,0;v)g(u)\|$$

$$+\|U(t,0;v)g(u) - U(t,0;v)g(v)\|$$

$$+\int_0^t \left[\left\| U(t,s;u)f\left(s,u(s),\int_0^s k\big(s,\tau,u(\tau)\big)\,d\tau,\int_0^a h\big(s,\tau,u(\tau)\big)\,d\tau\right)\right.\right.$$

$$\left. -U(t,s;v)f\left(s,u(s),\int_0^s k\big(s,\tau,u(\tau)\big)\,d\tau,\int_0^a h\big(s,\tau,u(\tau)\big)\,d\tau\right)\right\|$$

$$+\left\| U(t,s;v)f\left(s,u(s),\int_0^s k\big(s,\tau,u(\tau)\big)\,d\tau,\int_0^a h\big(s,\tau,u(\tau)\big)\,d\tau\right)\right.$$

$$\left.\left. -U(t,s;v)f\left(s,v(s),\int_0^s k\big(s,\tau,v(\tau)\big)\,d\tau,\int_0^a h\big(s,\tau,v(\tau)\big)\,d\tau\right)\right\|\right] ds$$

$$\leq C_1a\|u_0\|_Y \max\nolimits_{\tau\in J}\|u(\tau) - v(\tau)\| + GC_1a\max_{\tau\in J}\|u(\tau) - v(\tau)\| + M_0G_1\max\nolimits_{\tau\in J}\|u(\tau) - v(\tau)\|$$

$$+C_1a(F_1r + F_1K_1ar + F_1K_2a + F_1H_1ar + F_1H_2a + F_2)\max_{\tau\in J}\|u(\tau) - v(\tau)\|$$

$$+M_0F_1a\max_{\tau\in J}\|u(\tau) - v(\tau)\| + M_0F_1K_1a^2\max_{\tau\in J}\|u(\tau) - v(\tau)\|$$

$$+M_0F_1H_1a^2\max_{\tau\in J}\|u(\tau) - v(\tau)\|$$

$$\leq q\max\nolimits_{\tau\in J}\|u(\tau) - v(\tau)\|,$$

where

$$q = C_1 a\|u_0\|_Y + GC_1 a + M_0 G_1 + C_1 a(F_1 r + F_1 K_1 ar + F_1 K_2 a + F_1 H_1 ar + F_1 H_2 a + F_2) + M_0 F_1 a + M_0 F_1 K_1 a^2 + M_0 F_1 H_1 a^2$$

and hence, we obtain

$$\|Fu(t) - Fv(t)\| \le q \max_{\tau\in J}\|u(\tau) - v(\tau)\|,$$

with $0 < q < 1$. This shows that the operator F is a contraction on S. From the contraction mapping theorem it follows that F has a unique fixed point $u \in S$ which is the mild solution of (6) – (7) on $[0, a]$. Note that $u(t)$ is in $C(J, Y)$ by (E_6) (see [12] Lemma 7.4). In fact, $u(t)$ is weakly continuous as a Y-valued function. This implies that $u(t)$ is separably valued in Y, hence it is strongly measurable. Then, $\|u(t)\|_Y$ is bounded and measurable function in t. Therefore, $u(t)$ is Bochner integrable (see e.g. [17, Chapter-V]). Using relation $u(t) = Fu(t)$, we conclude that $u(t)$ is in $C(J, Y)$.

Consider the following linear evolution equation

$$v'(t) + B(t)v(t) = h(t),\ t \in [0, a] \tag{13}$$

$$v(0) = u_0 - g(u) \tag{14}$$

where $B(t) = A\big(t, u(t)\big)$ and $h(t) = f\left(t, u(t), \int_0^t k\big(t, s, u(s)\big)ds, \int_0^a h\big(t, s, u(s)\big)ds\right)$, $t \in [0, a]$ and u is the unique fixed point of F in S. We note that $B(t)$ satisfies $(H_1) - (H_3)$ of [15] (Section 5.5.3) and $h \in C(J, Y)$. Theorem 5.5.2 in Pazy [15] implies that there exists a unique function $v \in C(J, Y)$ such that $v \in C^1\big((0, a], X\big)$ satisfying (13) – (14) in X and v is given by

$$v(t) = U(t, 0; u)u_0 - U(t, 0; u)g(u) + \int_0^t U(t, s; u)\, f\left(s, u(s), \int_0^s k\big(s, \tau, u(\tau)\big)d\tau, \int_0^a h\big(s, \tau, u(\tau)\big)d\tau\right)ds, \qquad t \in J,$$

where $U(t, s; u), 0 \le s \le t \le a$ is the evolution system generated by the family $\{A\big(t, u(t)\big)\}$, $t \in J$, of linear operator in X. The uniqueness of v implies that $v \equiv u$ on J and hence u is a classical solution of (6) – (7) and $u \in C([0, a]: Y) \cap C^1\big((0, a]: X\big)$.

References

[1] D. Bahuguna, *Quasilinear integrodifferential equations in Banach spaces*, Nonlinear Anal. 24 (1995), 175-183.

[2] D. Bahuguna, *Regularity solutions to quasilinear integrodifferential equations in Banach spaces*, Appl. Anal. 62 (1996), 1-9.

[3] K. Balachandran and M. Chandrasekaran, *Existence of solution of a delay differential equation with nonlocal condition*, Indian J. Pure Appl. Math. 27 (1996), 443-449.

[4] K. Balachandran and S. Ilamaran, *Existence and uniqueness of mild and strong solutions of a semilinear evolution equation with nonlocal conditions*, Indian J. Pure Appl. Math. 25 (1994), 411-418.

[5] K. Balachandran and F.P. Samuel, *Existence of solutions for quasilinear delay integrodifferential equations with nonlocal conditions*, EJDE, 2009 (6) (2009), 1-7.

[6] L. Byszewski, *Theorems about the existence and uniqueness of solutions of a semilinear evolution nonlocal Cauchy problem*, J. Math. Anal. Appl. 162 (1991), 494-505.

[7] L. Byszewski, *Theorems about the existence and uniqueness of continuous solution of nonlocal problem for nonlinear hyperbolic equation*, Appl. Anal. 40 (1991), 173-180.

[8] L. Byszewski, *Uniqueness criterion for solution to abstract nonlocal Cauchy problem*, J. Appl. Math. Stoch. Anal. 162 (1991), 49-54.

[9] M.G. Crandall and P. Souganidis, *Convergence of difference approximations of quasilinear evolution equation*, Nonlinear Analysis 10 (1986), 425-445.

[10] M.B. Dhakne and K.D., Kucche, *Existence of mild solution of mixed Volterra-Fredholm functional integrodifferential equation with nonlocal condition*, Applied Mathematical Sciences, 5 (8) (2011), 359-366.
[11] S. Kato, *Nonhomogeneous quasilinear evolution equations in Banach spaces*, Nonlinear Anal. 9 (1985), 1061-1071.
[12] T. Kato, *Quasilinear equations of evolution, with applications to partial differential equations*, Lecture Notes in Mathematics 448 (1985), 25-70, Springer-Verlag, New York.
[13] H. Oka, *Abstract quasilinear Volterra integrodifferential equations*, Nonlinear Anal. 28 (1997), 1019-1045.
[14] H. Oka and N. Tanaka, *Abstract quasilinear integrodifferential equations of hyperbolic type*, Nonlinear Anal. 29 (1997), 903-925.
[15] A. Pazy, *Semigroup of Linear Operators and Applications to Partial Differential Equations*, Springer Verlag, New York, 1983.
[16] Shengli Xie, *Existence of solutions for nonlinear Mixed type integrodifferential functional evolution equations with nonlocal conditions*, Abstract and Applied Analysis 2012 (2012), 1-11.
[17] K. Yosida, *Functional Analysis*, Springer (1968).