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Schrödinger Differential Equation and Wave Pockets for Elementary Particles in the Minkowski Spaces

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Abstract

In this paper we presented the partial differential equations for the wave-pockets in the Minkowski 4-dimensional spaces, and their relationship with the Schrödinger equation for the elementary particles[7]. In this paper we show that the Schrödinger equation does not describe the propagation of a single wave-pocket of an elementary particle but to the stream of particles.
Because of that it has only a statistical meaning that can be applied to the stream of particles, and only its probabilistic interpretation to a single particle is well founded. That is, it is not a wave description of a single particle, but represents only its probabilistically determined position in a given space.

In physics and mathematics, Minkowski space (or Minkowski time-space [4]) is the mathematical setting in which Einstein's theory of special relativity is most conveniently formulated. In this setting the three ordinary dimensions of space are combined with a single dimension of time to form a four-dimensional manifold for representing a time-space.
In theoretical physics, Minkowski space is often contrasted with Euclidean space. While a Euclidean space has only spacelike dimensions, a Minkowski

space also has one timelike dimension. His famous article "Space and Time", begins with the following text: "The conceptions about time and space, which I hope to develop before you to-day, has grown on experimental physical grounds. Herein lies its strength. The tendency is radical. Henceforth, the old conception of space for itself, and time for itself shall reduce to a mere shadow, and some sort of union of the two will be found consistent with facts."
Differently from the pseudo-Euclidean Minkowski space [4], where time is imaginary and Euclidean three dimensions real, here we define the basic time-space four mutually orthogonal vectors $e_j, 0 \le j \le 3$, by the following matrix:

$$\begin{pmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{pmatrix}, \text{ with imaginary number } i = \sqrt{-1}.$$

Note that the matrix above is a particular case of the Minkowski tensor using a four-dimensional time-space, which combines the real dimension of time with the three imaginary dimensions of space.
Consequently, a vector of position in this space-time 4-dimensional system, w.r.t. a given referential coordinate system, is given by

$$\overrightarrow{\mathbf{r}_4} = cte_0 + xe_1 + ye_2 + ze_3 = cte_0 + \overrightarrow{\mathbf{r}},$$

where t is the time (i.e., ct is the timelike component of $\overrightarrow{\mathbf{r}_4}$, where c is the velocity of light in the vacuum) and $\overrightarrow{\mathbf{r}} = xe_1 + ye_2 + ze_3$ is an ordinary Euclidean vector with x, y, z three spatial coordinates.
Its infinitesimal amount is defined by $d\overrightarrow{\mathbf{s}} = cdte_0 + dxe_1 + dye_2 + dze_3$, where dt, dx, dy and dz are infinitesimal amounts of time-space dimensions.
Thus, in this 4-dimensional system the time is real while the three orthogonal space coordinates are imaginary. This choice is adopted in order to have that the distance

$$d\mathbf{s}^2 = d\overrightarrow{\mathbf{s}}d\overrightarrow{\mathbf{s}} = (cdt)^2 - dx^2 - dy^2 - dz^2,$$

for all local time-space reference systems of observations of quantum events be the positive real value (where space dimensions are limited).
Let us denote by $\overrightarrow{\mathbf{r}}_T(t) = x(t)e_1 + y(t)e_2 + z(t)e_3$ the vector that lies on the Euclidean 3-dimensional particle's trajectory. The 4-dimensional velocity of a given material point (tangent on its trajectory) in this Minkowski space is then defined by

$$\overrightarrow{\mathbf{v}_4} = \frac{d\overrightarrow{\mathbf{s}}}{dt} = ce_0 + v_xe_1 + v_ye_2 + v_ze_3 = ce_0 + \overrightarrow{\mathbf{v}},$$

where $\overrightarrow{\mathbf{v}} = \frac{\partial}{\partial t}\overrightarrow{\mathbf{r}}_T(t) = v_xe_1 + v_ye_2 + v_ze_3$ is the standard definition of the velocity in the 3-dimensional Euclidean space, with $v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}, v_z = \frac{dz}{dt}$ and

$v = |\overrightarrow{\mathbf{v}}| = \sqrt{v_x^2 + v_y^2 + v_z^2} \leq c$. The unitary 3-dimensional velocity vector is defined by $\overrightarrow{i_v} = \frac{\overrightarrow{\mathbf{v}}}{v}$.

Then we have that $|\overrightarrow{\mathbf{v}_4}| = \sqrt{c^2 - v_x^2 - v_y^2 - v_z^2} = c\sqrt{1-\beta^2}$, where $\beta = \frac{v}{c}$.

The trajectory of a material point that moves in the Minkowski 4-dimensional space by a velocity $\overrightarrow{\mathbf{v}_4}$ is defined by the *unitary tangent* vector of this trajectory $\overrightarrow{\tau} = \frac{\overrightarrow{\mathbf{v}_4}}{|\overrightarrow{\mathbf{v}_4}|} = \frac{\overrightarrow{\mathbf{v}_4}}{c\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\beta^2}} e_0 + \frac{v_x}{c\sqrt{1-\beta^2}} e_1 + \frac{v_y}{c\sqrt{1-\beta^2}} e_2 + \frac{v_z}{c\sqrt{1-\beta^2}} e_3$.

Then the vector $E_0\overrightarrow{\tau}$, where $E_0 = m_0c^2$ is the energy of the elementary particle with rest mass $m_0 > 0$, is the well defined *invariant* (independent of a referential system in special relativity theory) 4-dimensional *energy-momentum* vector for massive particles in the Minkowski spaces, that is,

$E_0\overrightarrow{\tau} = mc^2 e_0 + cmv_x e_1 + cmv_y e_2 + cmv_z e_3 = Ee_0 + c\overrightarrow{\mathbf{p}}$,

where $m = \frac{m_0}{\sqrt{1-\beta^2}}$ is the relativistic mass for a given velocity v of this particle, $E = mc^2$ is its total relativistic energy, and $\overrightarrow{\mathbf{p}} = m\overrightarrow{\mathbf{v}}$ is its 3-dimensional momentum.

Thus, both fundamental properties of a particle, its energy and its momentum, are two physical values that are propagated on the particle's trajectory with the velocity v. Consequently, the energy and momentum on the particle's trajectory are the functions that depend *only* on the time t.

An 4-dimensional angular wavenumber vector $\overrightarrow{\mathbf{k}_4}$, and its correspondent 4-dimensional momentum vector $\overrightarrow{\mathbf{p}_4}$, in this four-dimensional time-space are given by

$\overrightarrow{\mathbf{k}_4} = k_t e_0 + k_x e_1 + k_y e_2 + k_z e_3$, and $\overrightarrow{\mathbf{p}_4} = \hbar\overrightarrow{\mathbf{k}_4}$.

In what follows we will denote by $\overrightarrow{\mathbf{k}} = k_x e_1 + k_y e_2 + k_z e_3$ the spatial component of the angular wavenumber vector, with $k^2 = |\overrightarrow{\mathbf{k}}|^2 = k_x^2 + k_y^2 + k_z^2$, so that $k_4^2 = \overrightarrow{\mathbf{k}_4}\overrightarrow{\mathbf{k}_4} = k_t^2 - k^2$.

The mutually independent space-components are defined as usual by $k_x = \frac{2\pi}{\lambda_x}$, $k_y = \frac{2\pi}{\lambda_y}$, $k_z = \frac{2\pi}{\lambda_z}$, where $\lambda_x, \lambda_y, \lambda_z$ are spatial wavelengths w.r.t the axes x, y and z respectively, and $\lambda = \frac{2\pi}{k}$ is the (total) spatial wavelength. Let $\omega = 2\pi\nu$ be an angular frequency that depends on the space-components, $\nu = \frac{1}{T}$ with a time period T. Thus, $\lambda_t = cT$ is the time-like wavelength and $k_t = \frac{2\pi}{\lambda_t} = \frac{\omega(k)}{c}$ depends on the space-components in $\overrightarrow{\mathbf{k}_4}$, so that it holds that $dk_4 = dk = dk_x dk_y dk_z$, and $-\overrightarrow{\mathbf{k}_4}\overrightarrow{\mathbf{r}_4} = k_x x + k_y y + k_z z - \omega(k)t$, and $\overrightarrow{\mathbf{k}_4}\overrightarrow{\mathbf{v}_4} = \omega(k) + \overrightarrow{\mathbf{k}}\overrightarrow{\mathbf{v}}$.

Remark: from the relativistic theory we have that for each massive elementary particle (with rest mass m_0 greater than zero) it holds that $\omega(k) = \pm c\sqrt{k^2 + (m_0 c \setminus \hbar)^2}$, where $\hbar = \frac{h}{2\pi}$ is the Dirac's constant, for the Planck's

constant $h = 6.6210^{-34}Js$.
Note that we assume that ω can be positive or negative (clockwise or counter-clockwise angular frequency), so that the energy of the particle is $E = \hbar|\omega|$, where $|\_|$ denotes the absolute value. In the rest of this paper we will consider the cases when ω is positive.
Notice that if we represent an elementary particle, with energy $E = \hbar\omega$ and momentum $\overrightarrow{\mathbf{p}} = \hbar\overrightarrow{\mathbf{k}}$ [1], by a single harmonic $A\mathrm{e}^{-i\overrightarrow{\mathbf{k}_4}\overrightarrow{\mathbf{r}_4}}$ in this four-dimensional space, where $\overrightarrow{\mathbf{k}_4} = \frac{\omega}{c}e_0 + \overrightarrow{\mathbf{k}} = \frac{E}{\hbar c}e_0 + \frac{\overrightarrow{\mathbf{p}}}{\hbar}$, then we obtain that $k_4^2 = (\frac{\omega}{c})^2 - k^2 = 0$ for particles with rest mass equal to zero (photons, gravitons, etc..), and $k_4^2 > 0$ for the massive particles (with rest mass m_0 greater than zero). In fact, we obtain that $|k_4| = \frac{\omega_0}{c}$ where $\omega_0 = \frac{m_0c^2}{\hbar}$ is the invariant angular frequency for particles (analog to the invariant rest mass m_0 of particles). Thus, similarly to the 3-dimensional angular wavenumber vector $\overrightarrow{\mathbf{k}}$ that in physics means the particle's momentum, the 4-dimensional angular wavenumber $\overrightarrow{\mathbf{k}_4}$ has a physical meaning as the particle's relativistically invariant angular frequency.
□
The plan of this paper is the following: In Section 1 is introduced the concept for wave-pockets of matter-events in the Minkowski space, and are defined its integral expressions for the energy and momentum. Then in Section 2 are presented the definitions of partial differential equations of the first and the second order for these matter's wave-pockets. Finally, in Section 3 is elaborated the relationships with the Schrödinger differential equation of elementary particles and is demonstrated that it can be derived from differential equations for wave-pockets, defined in Section 2, as special case of the coherent dense stream of particles of the same type. Consequently, it is demonstrated that Schrödinger differential equation of elementary particles has only the well known *statistical* meaning if applied to a single elementary particle.

1 Introduction to wave-pockets of matter-events in the Minkowski space

In [8] was presented that in any given instance of time t, any matter-event in this time-space is a particular time-space perturbation $\Psi(\overrightarrow{\mathbf{r}_4})$, can be mathematically given by the following Fourier transformation:
$\Psi(\overrightarrow{\mathbf{r}_4}) = \Psi(x,y,z,t) = \int C(k_4)\mathrm{e}^{i(-\overrightarrow{\mathbf{k}_4}\overrightarrow{\mathbf{r}_4})}dk_4 =$
$= \int A(k)\mathrm{e}^{i(-\overrightarrow{\mathbf{k}}\overrightarrow{\mathbf{r}}-\omega(k)t)}dk =$ (where $A(k) = C(k_4) = C(\sqrt{(\frac{\omega(k)}{c})^2 - k^2}\ ))$

$= \int\int\int_{-\infty}^{+\infty} A(k)\mathrm{e}^{i(k_x x+k_y y+k_z z-\omega(k)t)} dk_x dk_y dk_z.$

It is a space-distribution of a particle in a given instance of time t, and it changes in time, that is, the amplitudes $A(k)$ are generally dependent on time as well.

Mathematically, these matter-events are complex functions, composed by one real and one imaginary component. The amplitudes $A(k)$ of the harmonics, in a given instance of time t, are given by inverse Fourier transformation,

$A(k) = \int\int\int_{-\infty}^{+\infty} \Psi(x,y,z,t)\mathrm{e}^{-i(k_x x+k_y y+k_z z-\omega(k)t)} dxdxdz.$

The elementary particles are pocket waves that propagate in this four-dimensional space.

Thus, for such particular *stationary* cases we have that $d\omega(k)/d\overrightarrow{\mathbf{k}}$ is constant (that is, it does not depend on the variable vector $\overrightarrow{\mathbf{k}}$), equal to the particle's velocity $-\overrightarrow{\mathbf{v}} = -v_x e_1 - v_y e_2 - v_z e_3$ (negative sign is the consequence that $e_i, i \geq 1$ are imaginary, thus the scalar products of (only) spatial vectors are negative), that can depend on the time t as well.

Consequently, for any fixed instance of time t, by integration we obtain that,

(0) $\int_{\overrightarrow{\mathbf{k}_0}}^{\overrightarrow{\mathbf{k}}} d\omega = \omega(k) - \omega(k_0) = -\int_{\overrightarrow{\mathbf{k}_0}}^{\overrightarrow{\mathbf{k}}} \overrightarrow{\mathbf{v}} d\overrightarrow{\mathbf{k}} = -\overrightarrow{\mathbf{v}} \int_{\overrightarrow{\mathbf{k}_0}}^{\overrightarrow{\mathbf{k}}} d\overrightarrow{\mathbf{k}} = -\overrightarrow{\mathbf{v}}(\overrightarrow{\mathbf{k}} - \overrightarrow{\mathbf{k}_0}),$

where the constant $\overrightarrow{\mathbf{k}_0} = \frac{\overrightarrow{\mathbf{p}}}{\hbar}$ for a given momentum $\overrightarrow{\mathbf{p}} = p_x e_1 + p_y e_2 + p_z e_3$ of a particle that is collinear with the velocity $\overrightarrow{\mathbf{v}}$, that is, $\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{v}} = -pv$. Because of that we can write $\overrightarrow{\mathbf{p}} = p\overrightarrow{i_v}$, $\overrightarrow{\mathbf{v}} = v\overrightarrow{i_v}$, where $\overrightarrow{i_v}$ is unitary vector tangent to the trajectory of a particle (i.e., $\overrightarrow{i_v}\overrightarrow{i_v} = -1$). Thus,

$\omega(k) = \omega_0 + v_x(k_x - \frac{p_x}{\hbar}) + v_y(k_y - \frac{p_y}{\hbar}) + v_z(k_z - \frac{p_z}{\hbar}),$

where ω_0 denotes the constant $\omega(k_0)$ that does not depend on k but may depend on time as we will see in what follows.

The *phase velocity* of a particle's pocket-wave, observed in a given referential system, is defined by $\vartheta = \frac{\omega_0}{k_0}$.

The constant ω_0 is determined as follows in the following two cases, by considering that the angular frequency $\omega(k)$ for its particular values is correlated by De Broglie to the total energy o particle $E = \hbar\omega(k)$:

- Case for *massive particles* (with rest mass $m_0 > 0$), denominated as mass-particles as well: when $\overrightarrow{\mathbf{v}} = 0$ then the energy of this particle is $E = m_0 c^2$, that is, the energy in the rest-state of this particle. Consequently, from (0) we have that $\omega_0 = \omega(k) = \frac{m_0 c^2}{\hbar}$.

  Consequently, for the total energy of these mass-particles that propagates with velocity $v = |\overrightarrow{\mathbf{v}}|$, with $\beta = \frac{v}{c}$, it holds that,

  $E = \sqrt{(m_0 c^2) + (pc)^2} = \sqrt{(\hbar\omega_0)^2 + (pc)^2} = \frac{m_0 c^2}{\sqrt{1-\beta^2}} = \hbar\omega_v,$

where $\omega_v = \omega_0/\sqrt{1-\beta^2}$ is a computed angular frequency relative to the velocity v of this particle w.r.t. the reference system of an observer (for different observes in different referential systems, that move with different velocities, this computed value of the *same* observed particle is different). For an observer in a given fixed position (the origin of its coordinate system, for example), this observed particle's frequency is by Lorentz low slowed down by the factor $\sqrt{1-\beta^2}$, so that the *really observed* particle's angular frequency of the observed wave-pocket $\Psi(\overrightarrow{\mathbf{r}_4})$ given above is constant and equal to $\omega_v\sqrt{1-\beta^2} = \omega_0$. Thus, ω_0 is the angular frequency of this massive particle equal in any inertial system (without acceleration), that is, an invariant as is the rest-mass m_0.

- Case for *massless particles* (with rest mass $m_0 = 0$): they propagate, as usual, with very high velocity $c \geq v > 0$ equal to the maximal velocity of light if this particle propagates in the vacuum, thus the zero value of equation (0) we can obtain when $\overrightarrow{\mathbf{k}} = \overrightarrow{\mathbf{k}}_0 = \frac{\overrightarrow{\mathbf{p}}}{\hbar}$. Consequently, the value of E is the *total* energy of this particle with the given momentum $\overrightarrow{\mathbf{p}}$, so that $\omega_0 = \omega(k) = \omega(\frac{p}{\hbar}) = \frac{E}{\hbar}$. The total energy of massless particles is defined by $E = \hbar\omega_o = (\hbar k_0)\vartheta = p\vartheta$. When a particle propagates in the vacuum then $\vartheta = c$, so that $E = pc$.
  When the total energy changes in time, to a fixed observer this angular frequency appears to change as well (for example, the relativistic effects for red-shifting of photons for a fixed observer). Thus, differently from massive particles where for a fixed observer $\frac{\partial\omega_0}{\partial t} = 0$, here ω_0 can change in time, if a particle changes its total energy during the propagation.

Consequently, for the wave-pocket of an elementary particle, and given reference system, we have that

$$(1) \quad \Psi(x,y,z,t) = \int A(k)\mathrm{e}^{i(-\overrightarrow{\mathbf{k}}\overrightarrow{\mathbf{r}}-\omega(k)t)}dk =$$
$$= (\int A(k)\mathrm{e}^{-i(\overrightarrow{\mathbf{k}}\overrightarrow{\mathbf{r}}-\overrightarrow{\mathbf{v}}(\overrightarrow{\mathbf{k}}-\overrightarrow{\mathbf{k}_0})t)}dk)\mathrm{e}^{-i\omega_0 t} =$$
$$= (\int A(k)\mathrm{e}^{-i(\overrightarrow{\mathbf{k}}-\overrightarrow{\mathbf{k}_0})(\overrightarrow{\mathbf{r}}-\overrightarrow{\mathbf{v}}t)}dk)\mathrm{e}^{i(-\overrightarrow{\mathbf{k}_0}\overrightarrow{\mathbf{r}}-\omega_0 t)} =$$
$$= \Phi(\overrightarrow{\mathbf{r}},t)\mathrm{e}^{i(-\frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{r}}}{\hbar}-\omega_0 t)} = \Phi(\overrightarrow{\mathbf{r}},t)\mathrm{e}^{i\frac{p}{\hbar}(-\overrightarrow{i_v}\overrightarrow{\mathbf{r}}-\vartheta t)}.$$

In what follows we introduce the spatial vector $\overrightarrow{\mathbf{u}} = \overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t$, that is equal to zero for the time-space points of particle's trajectory.
The "corpuscular" geometric wave-pocket shape (matter's distribution) of a particle, that *appears to a fixed observer*, is given by

$$\Phi(\overrightarrow{\mathbf{r}},t) = \Phi(x,y,z,t) = \int A(k)\mathrm{e}^{-i(\overrightarrow{\mathbf{k}}-\overrightarrow{\mathbf{k}_0})(\overrightarrow{\mathbf{r}}-\overrightarrow{\mathbf{v}}t)}dk =$$
$$= \int A(|\overrightarrow{\mathbf{k}} + \overrightarrow{\mathbf{k}_0}|)\mathrm{e}^{-i\overrightarrow{\mathbf{k}}\overrightarrow{\mathbf{u}}}dk \ =$$

(here we denote by $B(k)$ the value $A(|\overrightarrow{\mathbf{k}} + \overrightarrow{\mathbf{k}_0}|))$,
$= \int\int\int_{-\infty}^{+\infty} B(k)\mathrm{e}^{i(k_x(x-v_xt)+k_y(y-v_yt)+k_z(z-v_zt))}dk_xdk_ydk_z =$
$= \int\int\int_{-\infty}^{+\infty} B(k)\mathrm{e}^{i(k_xu_x+k_yu_y+k_zu_z)}dk_xdk_ydk_z$.
In the case when a particle propagates in the vacuum with a constant velocity $\overrightarrow{\mathbf{v}}$ (stationary case), then the coefficients $B(k)$ does not change in time, i.e. $\frac{\partial B(k)}{\partial t} = 0$, so that the "corpuscular" geometry (matter distribution) does not change in time and $\Phi(\overrightarrow{\mathbf{r}}, t) = \Phi(\overrightarrow{\mathbf{u}}) = \Phi(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t) = \Phi(x - v_xt, y - v_yt, z - v_zt)$ is a wave-pocket that propagates with a velocity $\overrightarrow{\mathbf{v}}$.
Thus, based on standard Fourier transformation, the function $\Phi(\overrightarrow{\mathbf{r}}, t)$ is a *real* function, differently from $\Psi(x, y, z, t$ that is *complex*. The real and imaginary components of $\Psi(x, y, z, t)$ are determined by the oscillation of the complex oscillator component $\mathrm{e}^{i(-\frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{r}}}{\hbar} - \omega_0 t)}$, that is an oscillation identical to the complex *plain wave* (like, for example, the complex electromagnetic plain wave). The amplitudes $B(k)$ of the harmonics can be obtained by the inverse Fourier transformation, for each given instance of time t, by:
$B(k) = \int\int\int_{-\infty}^{+\infty} \Phi(x, y, z, t)\mathrm{e}^{-i(k_xu_x+k_yu_y+k_zu_z)}dk_xdk_ydk_z$.
Thus, generally any particle is determined by the pocket-wave $\Psi(x, y, z, t)$ composed by two sub components: by the corpuscular matter distribution $\Phi(x, y, z, t)$ that is a real function, and by the 'phase wave' $\mathrm{e}^{i\varphi}$ that is a complex function of the particle's *phase* $\varphi = -\frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{r}}}{\hbar} - \omega_0 t$.
For any matter-perturbation of an elementary particle that propagates in the 3-dimensional space with a velocity that changes in the time, because of external forces that influence this particle, the 3-dimensional wave-pocket distribution $\Phi(x, y, z, t)$ changes as well, but it must satisfy the *conservation matter* principle, that is, at each given time instance t it must be satisfied the following *invariance* property:
(2) $\mathbf{1}_\Phi = \int\int\int_{-\infty}^{+\infty} \Phi(x, y, z, t)dxdydz = \int\int\int_{-\infty}^{+\infty} \Phi(x, y, z, 0)dxdydz > 0$,
where $\mathbf{1}_\Phi$ is a time-invariant constant value of an elementary particle (not necessarily equal to 1), and $V(t)$ is a finite cube (or sphere) which contains the whole "corpuscular" wave-pocket in a given instance of time t, and $dV = dxdydz$.
We define the minimal (limit) cube $V_m(t) = lim(2^3 \triangle X \triangle Y \triangle Z)$, such that in this time-instance t, $\Phi(x, y, z, t) = 0$ for ($x \leq - \triangle X$ or $x \geq \triangle X$ or $y \leq - \triangle Y$ or $y \geq \triangle Y$ or $z \leq - \triangle Z$ or $z \geq \triangle Z$).
The real function $\Phi(x, y, z, t)$ is the "corpuscular" geometric wave-pocket form of a particle that propagates in the ordinary 3-dimensional space with a velocity $\overrightarrow{\mathbf{v}} = v_xe_1 + v_ye_2 + v_ze_3$). In the stationary case, when it propagates in the vacuum with a constant velocity, it has constant distribution, that propagates

as pocket-wave $\Phi(\overrightarrow{\mathbf{u}}) = \Phi(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t) = \Phi(x - v_x t, y - v_y t, z - v_z t)$.
Analogously to the Schrodinger's approach, used to derive its equation based on total energies of particles, with the mapping $E \to i\hbar\frac{\partial}{\partial t}$, that does not take in consideration the spatial matter-distribution of a particle (considering in standard quantum theory only the pointlike particles), here, differently, we define the total energy E in a given time instance t, of the time-space perturbations defined by wave-pockets (1) and (2), by taking in consideration its real spatial matter distribution. Thus, by definition of a *spatial integral* as follows [6]:
(3) $E(t) = \ |\int_{-\infty}^{+\infty} i\hbar\frac{\partial\Phi(\overrightarrow{\mathbf{r}},t)\mathrm{e}^{-i\omega_0 t}}{\partial t}dV|/\mathbf{1}_\Phi = |\oint_{V_m(t)} i\hbar\frac{\partial\Phi(\overrightarrow{\mathbf{r}},t)\mathrm{e}^{-i\omega_0 t}}{\partial t}dV|/\mathbf{1}_\Phi$,
where $dV = dxdydz$.
Thus, the energy of the particles is and integral over Euclidean space, so it is only dependent on time in the 4-dimensional Minkowski space, so that $\omega_0 = \frac{E(t)}{\hbar}$ is only dependent on time (for massive particles it is a constant).
In [6] for the massive particles it was obtained that their momentum p is an integral over Euclidean space, thus only dependent on time, i.e.,
(4) $p(t) = \ |\oint_{V_m(t)}(-i\hbar\frac{\overrightarrow{\mathbf{v}}}{c}\nabla)\Phi(x - v_x t, y - v_y t, z - v_z t)dxdydz|/\mathbf{1}_\Phi$,
and, consequently, $k_0(t) = \frac{p(t)}{\hbar}$ as well.

2 Partial differential equations for wave-pockets

Let $\nabla = e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z}$ be the gradient, so that the Laplasian is defined by $\triangle = -\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. Then the derivation of the wave-pocket along its trajectory with the unitary tangent vector $\overrightarrow{i_v}$ on the trajectory, collinear with the vector of its velocity $\overrightarrow{\mathbf{v}} = v\overrightarrow{i_v} = \overrightarrow{i_v}\sqrt{v_x^2 + v_y^2 + v_z^2}$, is denoted by the operator $\overrightarrow{i_v}\nabla$. The propagation and geometric form of the wave-pocket $\Psi(x, y, z, t) = \Phi(\overrightarrow{\mathbf{r}}, t)\mathrm{e}^{i(-\frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{r}}}{\hbar} - \omega_0 t)}$, of an elementary particle that propagates with velocity $\overrightarrow{\mathbf{v}}$, are defined by the following differential equations [8]:
(e.1) $\mathrm{e}^{i\omega_0 t}\frac{\partial\Phi\mathrm{e}^{-i\omega_0 t}}{\partial t} = -i\frac{\partial(\omega_0 t)}{\partial t})\Phi + \overrightarrow{\mathbf{v}_1}\nabla\Phi + \Phi_D(\overrightarrow{\mathbf{r}}, t)$, where,
$\overrightarrow{\mathbf{v}_1} = \frac{\partial}{\partial t}(\overrightarrow{\mathbf{v}}t)$, $\Phi_D(\overrightarrow{\mathbf{r}}, t) = \int_{-\infty}^{+\infty}\frac{\partial B(k)}{\partial t}\mathrm{e}^{i(k_x(x - v_x t) + k_y(y - v_y t) + k_z(z - v_z t))}dk_x dk_y dk_z$ is equal to zero when this particle is in a stationary state, that is, when $\frac{\partial B(k)}{\partial t} = 0$.
(e.2) $\frac{\partial\Psi}{\partial t} = -i\omega_p\Psi + \overrightarrow{\mathbf{v}_1}\nabla\Psi + \Psi_D(\overrightarrow{\mathbf{r}}, t)$,
where $\Psi_D(\overrightarrow{\mathbf{r}}, t) = \Phi_D(\overrightarrow{\mathbf{r}}, t)\mathrm{e}^{i(-\frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{r}}}{\hbar} - \omega_0 t)}$.
(e.3) $(\frac{v_1}{c})^2\triangle\Psi - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} - ((\frac{\omega_p}{c})^2 + i\frac{1}{c^2}\frac{\partial\omega_p}{\partial t} - i\frac{\overrightarrow{\mathbf{v}_1}}{c^2\hbar}\frac{\partial\overrightarrow{\mathbf{p}}}{\partial t})\Psi - (i\frac{2\omega_p}{c^2}\overrightarrow{\mathbf{v}_1} - \frac{1}{c^2}\frac{\partial\overrightarrow{\mathbf{v}_1}}{\partial t})\nabla\Psi =$
$= \Upsilon_D(\overrightarrow{\mathbf{r}}, t)$,
where $\omega_p = \omega_1 + \frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{v}_1}}{\hbar}$, with $\omega_1 = \frac{\partial}{\partial t}(\frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{r}}}{\hbar} + \omega_0 t) = \frac{\overrightarrow{\mathbf{r}}}{\hbar}\frac{\partial\overrightarrow{\mathbf{p}}}{\partial t} + \frac{\partial}{\partial t}(\omega_0 t)$, $v_1 = \sqrt{-\overrightarrow{\mathbf{v}_1}\overrightarrow{\mathbf{v}_1}}$,

can change in time during the propagation. The right side of (e.3), $\Upsilon_D(\overrightarrow{\mathbf{r}}, t) = \frac{1}{c^2}(\overrightarrow{\mathbf{v}_1}\nabla\Phi_D - i2\omega_1\Phi_D + \frac{\partial\Phi_D}{\partial t})\mathrm{e}^{i(-\frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{r}}}{\hbar} - \omega_0 t)}$, is different from zero only in unstationary cases when $\Phi_D(\overrightarrow{\mathbf{r}}, t) \neq 0$.

In the *stationary* case, when ω_p and $\omega_1 = \omega_0$ are two constants, this equation can be given in a simpler D'Alambert-like form:

(e.4) $\quad (\frac{v_1}{c})^2\triangle\Psi_1 - \frac{1}{c^2}\frac{\partial^2\Psi_1}{\partial t^2} = -\frac{1}{c^2}\frac{\partial\overrightarrow{\mathbf{v}_1}}{\partial t}\nabla\Psi_1 - i\frac{\overrightarrow{\mathbf{v}_1}}{c^2\hbar}\frac{\partial\overrightarrow{\mathbf{p}}}{\partial t}\Psi_1,$

where $\Psi_1 = \mathrm{e}^{i\omega_p t}\Psi(\overrightarrow{\mathbf{r}}, t)$.

Notice that $\frac{\partial\omega_0}{\partial t} \neq 0$ only for the massless particles in their unstable states and very-very short interval of times $\triangle t \approx 0$, when they change their total energy $E = \hbar\omega_0$ during the collisions with another particles (the Compton effects). In what follows we will denote by $\mathcal{E} = \hbar\omega_p$ the energy associated to this angular frequency ω_p.

The *stationary case* is obtained when a particle propagates with constant total energy E and constant value $\mathcal{E} = \hbar\omega_p$. Thus, in such a stationary case we have that $\Phi_D(\overrightarrow{\mathbf{r}}, t) = 0, \Psi_D(\overrightarrow{\mathbf{r}}, t) = 0$. It is easy to verify that the stationary case is one, for example, of the following two cases:

1. When a particle propagates with constant momentum $\overrightarrow{\mathbf{p}}$, velocity $\overrightarrow{\mathbf{v}}$ (thus, $\frac{\partial\overrightarrow{\mathbf{v}}}{\partial t} = \frac{\partial\overrightarrow{\mathbf{p}}}{\partial t} = 0$), and total energy E (in that case ω_0 is constant for massless particles as well). Thus, without any acceleration. In that case ω_p is constant as well, with constant $\mathcal{E} = \hbar(\frac{\overrightarrow{\mathbf{r}}}{\hbar}\frac{\partial\overrightarrow{\mathbf{p}}}{\partial t} + \frac{\partial}{\partial t}(\omega_0 t) + \frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{v}_1}}{\hbar}) = \hbar\omega_0 + \overrightarrow{\mathbf{p}}\,\overrightarrow{\mathbf{v}} = \hbar\omega_0 - pv$.

2. When a particle routes with constant radius R around a fixed center, with constant angular velocity $\nu = \frac{|\overrightarrow{\mathbf{v}}|}{R} = \frac{v}{R}$, constant value of the momentum $p = |\overrightarrow{\mathbf{v}}|$, and total energy E. In this case we can obtain ω_p constant in a particular coordinate system: coordinate center of the reference system x, y of the plain in which this particles routes. Then, the position of the trajectory of this particle in a given moment t is equal to $\overrightarrow{\mathbf{r}} = R\overrightarrow{i_\theta}$, where $\overrightarrow{i_\theta}$ is a unitary radial vector with angle $\theta = \nu t$ w.r.t the axis x. In this case the acceleration $\frac{\partial\overrightarrow{\mathbf{v}}}{\partial t} = -|\frac{\partial\overrightarrow{\mathbf{v}}}{\partial t}|\overrightarrow{i_\theta}$, and $\frac{\partial\overrightarrow{\mathbf{p}}}{\partial t} = -|\frac{\partial\overrightarrow{\mathbf{p}}}{\partial t}|\overrightarrow{i_\theta}$ are radial vectors that have the constant values and are orthogonal to the vectors of velocity $\overrightarrow{\mathbf{v}}$ and momentum $\overrightarrow{\mathbf{p}}$. So that, $\overrightarrow{\mathbf{p}}\,\overrightarrow{\mathbf{r}} = 0$, $\overrightarrow{\mathbf{p}}\frac{\partial\overrightarrow{\mathbf{v}}}{\partial t} = 0$, and
   $\omega_p = \frac{\partial}{\partial t}(\frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{r}}}{\hbar} + \omega_0 t) + \frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{v}_1}}{\hbar} = +\omega_0 + \frac{1}{\hbar}\overrightarrow{\mathbf{p}}(\overrightarrow{\mathbf{v}} + t\frac{\partial\overrightarrow{\mathbf{v}}}{\partial t}) = \omega_0 + \frac{1}{\hbar}\overrightarrow{\mathbf{p}}\,\overrightarrow{\mathbf{v}} =$
   $= \omega_0 - \frac{pv}{\hbar}$.
   Here both ω_0 and pv are constant, and, consequently, we obtained the stationarity condition $\frac{\partial\omega_p}{\partial t} = 0$ in all points of the trajectory of this particle, with constant $\mathcal{E} = \hbar\omega_0 + \overrightarrow{\mathbf{p}}\,\overrightarrow{\mathbf{v}} = \hbar\omega_0 - pv$.

Notice that in both stationary cases above we obtained that this particular constant energy is equal to $\mathcal{E} = \hbar\omega_p = \hbar\omega_0 + \overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{v}} = \hbar\omega_0 - pv$. If $\omega_p = \omega_1 + \frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{v}_1}}{\hbar}$ is computed for current space-time positions on the particle's trajectory, then, in this particular case, ω_1 is taken as a derivation $\frac{\partial}{\partial t}$ of the current particle's phase $\varphi = -\frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{r}}}{\hbar} - \omega_0 t$ (we have that $\Psi(\overrightarrow{\mathbf{r}}, t) = \Phi(\overrightarrow{\mathbf{r}}, t)\mathrm{e}^{i\varphi}$) and express the particle's phase-changing on its trajectory.
Notice that the energy changes only during collisions with another particles (Compton effects; we consider a "field" as a statistical result of the iterations with bosons of this particular field), so that after it this particle continue to propagate again as a stationary particle, but with new values of total energy E, velocity $\overrightarrow{\mathbf{v}}$, momentum $\overrightarrow{\mathbf{p}}$, and new stable wave-pocket geometry (distribution) $\Phi(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t)$, so that it can be described by the simpler stationary-case differential equations:

(e.1.1) $\quad \mathrm{e}^{i\omega_0 t}\frac{\partial \Phi \mathrm{e}^{-i\omega_0 t}}{\partial t} = (-i\omega_0 + \frac{\partial(\overrightarrow{\mathbf{v}}t)}{\partial t}\nabla)\Phi$

(e.2.1) $\quad \frac{\partial \Psi}{\partial t} = -\frac{i}{\hbar}\mathcal{E}\Psi + \frac{\partial(\overrightarrow{\mathbf{v}}t)}{\partial t}\nabla\Psi,$

where, when a velocity $\overrightarrow{\mathbf{v}}$ is constant, we have that $\frac{\partial(\overrightarrow{\mathbf{v}}t)}{\partial t} = \overrightarrow{\mathbf{v}} + t\frac{\partial \overrightarrow{\mathbf{v}}}{\partial t} = \overrightarrow{\mathbf{v}}$.
Notice that we have particular stationary cases when $\overrightarrow{\mathbf{v}}$ is not constant, as for instance, for a stationary electron that rotates around the nucleus of an atom with a constant radial acceleration (in that case $\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{v}}$ and total energy of this electron are constant, thus $\mathcal{E} = E + \overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{v}}$ is constant as well).
It was shown [6] that, in the case of these massless elementary particles that propagate in vacuum with the velocity of light c, we have that this "corpuscular" wave pocket in the *stable*(general) state corresponds to the Dirac function (with $A(k) = B(k) = 1/(2\pi)^3$), $\quad \delta(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{c}}t) = \delta(x - c_x t, y - c_y t, z - c_z t) = \delta(x - c_x t)\delta(y - c_y t)\delta(z - c_z t)$. It is reasonable assumption that the volume of a distribution Φ (where it is greater than zero) of a massive particle (with rest mass m_0 greater than zero) is always greater than zero, so that it is a reason that such particles can not reach the limit velocity of light. In the analog way, the massless particles (with rest mass equal to zero) must have, in their stable state, this volume equal to zero, so that their distribution Φ is equal to Dirac function above, and they are able to propagate with the velocity of light in the vacuum. The non stable states of particles with rest mass $m_0 = 0$ can have more complex wave-pocket forms and it happen only in a very short instances of time, when the particle enters in strongly unsymmetric space region, as will be explained in what follows. In such situations its velocity of propagation becomes less than the velocity of light in the vacuum so that this particle can have a similar behavior as massive particles with Φ that occupies a limited but

nonzero volume (so called spatial explosion of excited bosons). This unstable state of the particles with $m_0 = 0$ tends to come back into the stable state with Dirac function geometry for distribution Φ.

The interactions between any two pocket-waves (particles) can be obtained only by their local collisions, and depending on their energy and velocities they can produce a kind of Compton effects (elastic collisions) where they survive the collisions by chaining their momentum and energy (with conservation of total momentum and energy), or can make total fusion between them with possible creation of new stable particles (in Feynman's diagrams). In order to be able for two pocket-waves to have a collision, and mutual interference, at least one of them must have a volume $V_m(t)$ (in a given instance of time of mutual collision) greater than zero. So, from this point of view, it can not happen that the distance between any two particles becomes equal to zero, so that we avoid classic infinitary problems of gravitational and electronic fields and forces where the particles are pointlike, so that it is possible to have the distances between particles equal to zero with, consequently, infinite values of gravitational (or electric) forces.

The particles with $V_m(t)$ equal to zero are, for example, the particles with $\Phi(x-v_xt, y-v_yt, z-v_zt)$ equal to the Dirac function $\delta(x-v_xt, y-v_yt, z-v_zt) = \delta(x-v_xt)\delta(y-v_yt)\delta(z-v_zt)$.

Thus, for any two particles with the "corpuscular" form given by Dirac function, it is impossible to have the collisions in their stable states, but only when they are excited and are involved in their temporary "spatial explosions". Such explosions can happen also when two stable particles are very close one to another so that the ideal spatial symmetry for a free particle in the vacuum does not hold more: it explains why, for example, photons can interact with gravitons (i.e., gravitational field) and may have the gravitational redshifts. Because of that, it will be natural consequence that the massless particles, as bosons (gravitons, photons, etc..) in their stable states, will have the volume $V_m(t)$ equal to zero (with Dirac function for their distribution Φ). In that case they can be used as intermediators between the massive particles (that have the rest mass and the volume $V_m(t)$ greater than zero), that is, to be the quantum-correspondence for the "fields" (the statistical events as gravitational, electromagnetic, etc., that are statistical results of actions of a high number of bosons), by avoiding in more common situations the significant interference between themselves.

In the *stationary cases*, the basic equation (e.4) can be divided into following cases:

- When the velocity $v = 0$, we obtain a simple equation:
(e.4.0) $\frac{\partial^2\Psi_1}{\partial t^2} = 0$, that is, $\frac{\partial^2\Psi}{\partial t^2} = -\omega_0^2\Psi$,
with a simple solution, $\Psi(x, y, z, t) = \Phi(x, y, z)\mathrm{e}^{-\omega_0 t}$.

- When the velocity $\overrightarrow{\mathbf{v}}$ and the momentum $\overrightarrow{\mathbf{p}}$ are constant vectors during the propagation, different from zero, then $\overrightarrow{\mathbf{v}_1} = \overrightarrow{\mathbf{v}}$, so we obtain D'Alambert equation where v is a constant value:
(e.4.1) $\triangle\Psi_1 - \frac{1}{v^2}\frac{\partial^2\Psi_1}{\partial t^2} = 0$,
with the solution $\Psi_1(x, y, z, t) = \Phi(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t)\mathrm{e}^{-i\frac{\overrightarrow{\mathbf{p}}}{\hbar}(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t)}$, thus,
$\Psi(x, y, z, t) = \Psi_1\mathrm{e}^{-\omega_p t} = \Phi(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t)\mathrm{e}^{i(-\frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{r}}}{\hbar} - \omega_0 t)}$.

- The case when the velocity $\overrightarrow{\mathbf{v}}$ with $v > 0$ and the momentum $\overrightarrow{\mathbf{v}}$ change only the direction (they are collinear vectors in each instance of time) during a propagation, but not their values, so that $-\frac{\partial(\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{v}})}{\partial t} = \frac{\partial(pv)}{\partial t} = 0$ and $v_1 = v$ is constant as well (this case includes the second case in Example 2 as well: when the velocity of massive particle with rest mass m_0 is $\overrightarrow{\mathbf{v}} = v(-\cos\theta e_1 + \sin\theta e_2)$, where $\theta = \nu t = \frac{v}{R}t$ is the angle w.r.t the axis x of a particle that routes around the coordinate center with constant angular velocity $\nu = \frac{v}{R}$ on circular orbit with a radius R).
Thus, from (e.4) we obtain an extended D'Alambert equation where $v^2 = |\overrightarrow{\mathbf{v}}\overrightarrow{\mathbf{v}}| > 0$ is a constant value: (e.4.2) $\triangle\Psi_1 - \frac{1}{v^2}\frac{\partial^2\Psi_1}{\partial t^2} = \Theta(\overrightarrow{\mathbf{r}}, t)$,
where $\Theta(\overrightarrow{\mathbf{r}}, t) = -\frac{1}{v^2}(\frac{\partial\overrightarrow{\mathbf{v}_1}}{\partial t}\nabla\Psi_1 + i\frac{\overrightarrow{\mathbf{v}_1}}{\hbar}\frac{\partial\overrightarrow{\mathbf{p}}}{\partial t}\Psi_1)$. Thus, we obtain a general solution $\Psi_1(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t) = \Phi(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t)\mathrm{e}^{-i\frac{\overrightarrow{\mathbf{p}}}{\hbar}(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t)}$.

Notice that the massive particles that does not change the total energy E during a propagation are always in the stationary states. In all cases, the geometric form (the distribution Φ) of a particle in a given time-instance t depends on the particular boundary conditions for the differential equations as well. In the case when they are far from another massive particles (usually it can be considered if another particles are far from this particle in order of one millimeter), then Φ is symmetric w.r.t. the direction of propagation. Otherwise, the boundary conditions for these differential equations can drastically change, with the result that Φ can become enormously bigger than in normal situations, that is, they can instantaneously "explode", because the single harmonics of the Fourier representation of $\Phi(x, y, z)$ in a given time-instance t are contemporarily present in the whole 3-dimensional Euclidean space.
Let us consider, for example, the case $m_0 = 0$ of particles with rest mass equal to zero:
In their stable state, enough far from another particles, they have the

Dirac function for geometric wave-packet form [6], $\Psi(x,y,z,t) = \delta(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t)\exp^{i(-\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{r}}-Et)/\hbar}$.
But there are the situations when a stable, stationary, photon becomes excited for a short interval of time, as in the situations when is sharply broken the *space symmetry* during its propagation(thus, the boundary conditions for the differential equations of particle's propagation are drastically changed). In all these situations a photon may change its momentum, direction of propagation and its velocity, without changing its total energy, because these "interactions" are not based on collisions with another particles (as Compton effects, or fusions), but on instantaneous space explosions of their geometric wave-pocket form Φ (which is the zero-volume Dirac function id their stable states) caused by an *instantaneous* changing of the amplitudes $B(k)$ of its wave-pocket harmonics $B(k)\exp -i\overrightarrow{\mathbf{k}}(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t)$ in the presence of a local sharply broken space symmetry. These ate typical cases for the particle's "*explosions*", when we take in consideration the general equations (e.1) and (e.2) for movements of particles where the component $\Phi_D(\overrightarrow{\mathbf{r}},t)$ is dominant, caused by the fact that in such dynamic framework we have that $\frac{\partial B(k)}{\partial t} \neq 0$ caused by a dynamical changing the boundary conditions in the local space around this particle.

3 Stream of particles: statistical meaning of Schrödinger equation

It is well known that the relativistic version of the Schrödinger equation for the elementary particles is postulated by the following Klein-Gordon second-order differential equation:
(e.7) $\triangle\psi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi = (\frac{\omega_0}{c})^2\psi$,
whose solution can be given by $\psi = \varphi(x,y,z,t)\mathrm{e}^{-i\omega_0 t}$.
The solution for a propagation of the elementary particles in the stationary cases, when ω_0 and ω_p are constant in time, presented previously, can be postulated in the similar way, by using the particle's second-order D'Alambert-like differential equation:
(e.4) $(\frac{v_1}{c})^2\triangle\Psi_1 - \frac{1}{c^2}\frac{\partial^2\Psi_1}{\partial t^2} = -\frac{1}{c^2}\frac{\partial\overrightarrow{\mathbf{v}_1}}{\partial t}\nabla\Psi_1 - i\frac{\overrightarrow{\mathbf{v}_1}}{c^2\hbar}\frac{\partial\overrightarrow{\mathbf{p}}}{\partial t}\Psi_1$,
where $\Psi_1 = \mathrm{e}^{i\omega_p t}\Psi(\overrightarrow{\mathbf{r}},t)$,
with $\Psi_1 = \mathrm{e}^{i\omega_p t}\Psi$, where $\Psi(x,y,z,t)$ describes an elementary particle that propagates with a velocity $\overrightarrow{\mathbf{v}}$ and a momentum $\overrightarrow{\mathbf{p}}$, and, possibly, with an acceleration $\frac{\partial\overrightarrow{\mathbf{v}}}{\partial t}$, such that $\omega_p = \frac{E+\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{v}}}{\hbar} = \frac{E-pv}{\hbar}$, where E is the potential energy of this particle (equal to m_0c^2) if $m_0 > 0$, and total energy otherwise.

Then, the solution of (e.4) is $\Psi_1(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t) = \Phi(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t)\mathrm{e}^{i\frac{\overrightarrow{\mathbf{p}}}{\hbar}(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t)}$, where real function $\Phi(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t)$, at any fixed time instance t is a geometric distribution of this particle in that moment. It depends on the boundary conditions for the differential equation (e.4) for this particle in this given time-instance t. If this particle propagates in the vacuum, than Φ is symmetric w.r.t the direction of propagation, but if in a given moment t this particle propagates nearly to another massive particles, than the boundary conditions change and depend on the spatial presence (their distributions) of these particles. Consequently, the geometric distribution of a massive particle can change (see the equation (e.3) for the general case) also when it propagates with constant velocity $\overrightarrow{\mathbf{v}}$, in presence of another massive particles.

This fact is very important when a particle that propagates in the vacuum, encounter some material obstacle, so that Φ, in the momentum when is broken this space symmetry, can drastically change and increment its volume, in a short interval of time. It is important to denote that such a changing of the geometric distribution is instantaneous in all points of the space, and has no any constraint as, for example, the maximal velocity of energy transportation, that is equal to the velocity of light from relativistic theory of Einstein.

From the fact that $\Psi_1 = \mathrm{e}^{i\omega_p t}\Psi$, we obtain that any elementary particle has the following wave-pocket geometric form (that was previously postulated in (1)):

$\Psi(x, y, z, t) = \Phi(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{v}}t)\mathrm{e}^{i(-\frac{\overrightarrow{\mathbf{p}}\overrightarrow{\mathbf{r}}}{\hbar} - \omega_0 t)}$

In the case when the velocity of propagation is constant, equal to zero, then we obtain the solution $\Psi(x, y, z, t) = \Phi(x, y, z)\mathrm{e}^{-i\omega_0 t}$, where $\omega_0 = \frac{E}{\hbar}$ and E is the potential energy of this particle.

There is the following relationship between Klein-Gordon and our differential equations:

Proposition 1 *The Klein-Gordon equation corresponds to the differential equation (e.4) of an elementary particle if its velocity of propagation is equal to zero.*

Proof: In the case when $\overrightarrow{\mathbf{v}} = 0$ and $\frac{\partial \overrightarrow{\mathbf{v}}}{\partial t} = 0$, so that $\omega_p = \omega_0$ is constant, then, from (e.4), we obtain that:

$0 = \frac{\partial^2 \Psi_1}{\partial t^2} =$

$= (-\omega_0^2 \Psi + \frac{\partial^2 \Psi}{\partial t^2} + i2\omega_0 \frac{\partial \Psi}{\partial t})\mathrm{e}^{i\omega_0 t} =$

$= (-\omega_0^2 \Psi + \frac{\partial^2 \Psi}{\partial t^2} + i2\omega_0(\overrightarrow{\mathbf{v}}\nabla\Psi - i\omega_0\Psi))\mathrm{e}^{i\omega_0 t} =$

$= (\omega_0^2 \Psi + \frac{\partial^2 \Psi}{\partial t^2})\mathrm{e}^{i\omega_0 t}$.

Thus, we obtain the equation

(e.8) $\frac{\partial^2\Psi}{\partial t^2} = -\omega_0^2\Psi$,
that is equal to Klein-Gordon equation when $\triangle\psi = 0$, that is, when the momentum $p = 0$, and consequently, the velocity is equal to zero, and with the solution $\Psi(x, y, z, t) = \Phi(x, y, z)\mathrm{e}^{-i\omega_0 t}$.
Then we have that it holds the Schrödinger equation for the total energy, $E\Psi = i\hbar\frac{\partial\Psi}{\partial t}$, and the total energy formula (3) introduced in this paper,
$E = \ |\int i\hbar\frac{\partial\Phi(x-v_xt, y-v_yt, z-v_zt)\mathrm{e}^{-i\omega_0 t}}{\partial t}dV|/\mathbf{1}_\Phi =$
$= |\int i\hbar\frac{\partial\Phi(x,y,z)\mathrm{e}^{-i\omega_0 t}}{\partial t}dV|/\mathbf{1}_\Phi =$
$= |\int i\hbar(-i\omega_0)\Phi(x, y, z)\mathrm{e}^{-i\omega_0 t}dV|/\mathbf{1}_\Phi =$
$= \hbar\omega_0(\int \Phi(x, y, z)dV)/\mathbf{1}_\Phi = \hbar\omega_0,$
from the fact that for the particles is satisfied the normalization principle (2), $\mathbf{1}_\Phi = \int \Phi(x, y, z)dV$.
□
But, if the velocity of a particle is equal to zero, then we can not have any phenomena of the plain waves, and as we will see, in the interesting cases when $\overrightarrow{\mathbf{v}}$ is different from zero, the equation (e.4) of propagation of particles is completely different from the equation Klein-Gordon, and consequently, from the Schrödinger equation.
The wave-particle-duality, the fundamental component of the new quantum formalism in Bohrs opinion, was reformulated by incorporating the results of some experiments accomplished in the last decades of twentieth century.
The Bohrs complementarity principle stated the mutual exclusiveness and joint full completeness of the two (classical) descriptions of quantum systems; after Einstein-Podolsky-Rosenpaper, the wave-particle duality, or wave-particle complementarity, could be expressed by stating that it is impossible to build up an experimental arrangement in which we observe at the same time both corpuscular and wave aspects. In a two-slit experiment, they would correspond, respectively, to the which-way knowledge and the observation of interference pattern. Bohr showed this mutual exclusivity in numerous examples [5], and linked it to the unavoidable disturbance inherent in any measurement event.
Not everyone agreed with this interpretation, or with Born and Heisenberg's statement about wave-particle-duality. Einstein and Schrödinger were among the most notable dissenters. Until the ends of their lives they never fully accepted the Copenhagen doctrine. Einstein was dissatisfied with the reliance upon probabilities. But even more fundamentally, he believed that nature exists independently of the experimenter, and the motions of particles are precisely determined. It is the job of the physicist to uncover the laws of nature that

govern these motions, which, in the end, will not require statistical theories. The fact that quantum mechanics did seem consistent only with statistical results and could not fully describe every motion was for Einstein an indication that quantum mechanics was still incomplete.

Recently it was demonstrated that intermediate particle-wave behaviors exist and, in addition to that, there are single experiments in which both classical wave-like and particle-like behaviors are showed total and simultaneously on an individual system [2]. For instance, in the Boses double-prism experiment , tunneling and perfect anticoincidence were observed in single photon states.

Consequently, the meaning of the wave-particle duality must incorporate the simultaneous use of the two classical descriptions in the interpretation of experiments, loosing their original mutual exclusivity , which is incorporated as an extreme case in the new interferometric duality, a continuous quantum concept.

In fact, our results demonstrate that Ψ of any elementary particle is always a pocket-wave (with a "corpuscular" space-geometric distribution $\Phi(x, y, z, t)$) delimited in the space at each time instance, but that has contemporarily an oscillation in the complex space ('phase wave'), expressed by its complex component $e^{i(-\frac{\vec{p}\vec{r}}{\hbar}-\omega_0 t)}$.

Now we will show that each relatively dense stream of particles of the same type, velocity, energy and direction of propagation compose a complex plain wave, as, for example, an electromagnetic plain wave when these particles are photons.

Proposition 2 *Any dense stream of elementary particles of the same type, velocity, energy and direction of propagation compose a perfect plain wave.*

Proof: Let us suppose a stream of particles that propagates along the x axis, with the distance between two consecutive particles in this stream of particles is $\triangle x << \lambda$, where $\lambda = \frac{2\pi\hbar}{p}$ and p is the momentum of each particle.

Then we have that this stream of particles in the moment $t = N\triangle t$, $N >> 1$ (where $n = N$ is the last emitted particle in the source at $x = 0$, and $n = 0$ is the first emitted particle from this source), is equal to:

$\sum \Psi_n(x, y, z, t) = \sum_{n=0,1,..,N} \Phi_n(x - v(t - n\triangle t)e^{i(\frac{2\pi}{\lambda}v(t-n\triangle t)-\omega_0(t-n\triangle t))} =$

$= \sum_{n=0,1,..,N} \Phi_n(x - (N - n)v\triangle t)e^{i(\frac{2\pi}{\lambda}v(N-n)\triangle t-\omega_0(N-n)\triangle t)} =$

$= \sum_{j=N,N-1,..,0} \Phi_n(x - jv\triangle t)e^{i(\frac{2\pi}{\lambda}vj\triangle t-\omega_0(j\triangle t)}$,

Thus in the limit case when $\triangle t \mapsto 0$, we obtain that:

$lim_{\triangle t\mapsto 0}Re(\sum \Psi_n(x, y, z, t)) = ARe(\int_{\alpha N\triangle t}^{0} e^{i\alpha t'} d\alpha t') =$

(where $\alpha = \frac{2\pi}{\lambda} v - \omega_0$, $t' = j \triangle t$)
$= A \int_{\alpha N \triangle t}^{0} \cos(\alpha t') d\alpha t' =$
$= A \sin(\frac{2\pi}{\lambda} vt - \omega_0 t) =$ (from $t = N \triangle t$)
$= A \sin(\frac{2\pi}{\lambda} x - \omega_0 t) = A \sin(\frac{p}{\hbar} x - \omega_0 t)$,
where A is a constant proportional to the density of this stream of particles. Thus, this stream of particles is represented by the plain wave $A \sin(\frac{2\pi}{\lambda} x - \omega_0 t)$, with wave-length λ and angular frequency ω_0.
□
The non dense streams will compose a kind of amplitude-modulated complex plain waves.
Consequently, it is valid the following lemma:

Lemma 1 *The Schrödinger equation can be obtained by application of the energy conservation over a complex plain wave $A e^{i(\frac{p}{\hbar} x - \omega_0 t)}$ of a given stream of particles.*

Proof: Let us consider the stream $\psi = A e^{i(\frac{p}{\hbar} x - \omega_0 t)}$ of the massive particles, where $\omega_0 = \frac{m_0 c^2}{\hbar}$. If we apply the relativistic energy equation for the elementary particles (the stream is composed by a finite number of them) $E^2 = (m_0 c^2)^2 + (cp)^2$ to this plain wave, we obtain the Klein-Gordon equation:
(e.7) $\quad \triangle \psi - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \psi = (\frac{\omega_0}{c})^2 \psi$,
whose solution can be given by $\psi = \phi(x, y, z, t) e^{-i\omega_0 t}$, with $\triangle \psi = e^{-i\omega_0 t} \triangle \phi$, so that we obtain the first and second time derivation:
$\frac{\partial \psi}{\partial t} = (-i\omega_0 \phi + \frac{\partial \phi}{\partial t}) e^{-i\omega_0 t}$,
$\frac{\partial^2 \psi}{\partial t^2} = (-\omega_0^2 \phi - i 2\omega_0 \frac{\partial \phi}{\partial t}) e^{-i\omega_0 t}$,
by assuming that $\frac{\partial^2 \phi}{\partial t^2}$ is infinitesimal. Then, if we substitute this last equation into (e.7), we obtain the Schrödinger equation in the absence of potential [3]:
(e.9) $\quad \triangle \phi + i \frac{2m_0}{\hbar} \frac{\partial}{\partial t} \phi = 0$.
□
It is easy to see that the Klein-Gordon equation (e.7) is very different from the equations (e.3) and (e.4) for a propagation of a single massive particle, if its velocity v is different from zero.
In order to reduce the stationary case (e.4) to Klein-Gordon equation, and, consequently, to Schrödinger equation, it would be necessary that this particle satisfies the following conditions: to propagate with velocity of light in the vacuum $v = c$, to have $\omega_p = 0$, that is $\omega_0 = \frac{pc}{\hbar}$, and with $-i \frac{\vec{\mathbf{c}}}{\hbar} \frac{\partial \vec{\mathbf{p}}}{\partial t} = \omega_0^2$ (while $\frac{\partial \vec{\mathbf{v}}}{\partial t} = 0$ for the real component of the right part of equation (e.4)), that is impossible.

4 Conclusion

Thus, Schrödinger equation in any case can not represent the *propagation* of a single particle. Thus, we have to deal with the reasonable doubts about some of the previously obtained results for the *single* particles, that are based on the deductions from the Schrödinger equation. Especially when they result with fundamentally non classical characters of a quantum state of a particle(as for example in the case of the Wigner function for a single photon with *negative* probabilities).

Consequently, the Schrödinger equation obtained by application of the energy conservation over a complex plain wave $Ae^{\frac{p}{\hbar}x-\omega_0 t}$, corresponds to its application not to a single particle but to the complete stream of particles.

Because of that it has only a statistical meaning that can be applied to the stream of particles, and only its probabilistic interpretation to a single particle is well founded. That is, it is not a wave description of a single particle, but represents only its probabilistically determined position in a given space.

From this point of view, the current interpretation of Schrödinger equation is confirmed, and explains its utility, but it has only a statistical meaning, and its non-determinism is only the consequence of this statistical meaning applicable to single particles.

The underlying particle's theory, presented in this paper demonstrates that Einstein's idea was correct, and that the differential equations derived previously in this paper demonstrate their deterministic and classical corpuscular nature. Only the streams of these particles in the determined conditions (the equal energy, momentum and direction of propagation) result in the simple plain waves.

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