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Double-slit Experiment: a Test for Individual Particles Completion of Quantum Mechanics

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Abstract

We consider the *3-dimensional* (3-D) model of the massive particles, represented as the rest-mass energy-density wave-packets, which is analog to the common physical objects which we experiment in our every-day life. This new theory is a completion of the statistical QM (where the wavefunctions represent the ensemble of measurements) by the QM theory for the individual particles. Then we provide an analysis of particle internal dynamic during their strong excitations caused by the sharp breaking of the 3-D vacuum symmetry during particle inertial propagation. Hence for that we use the three conservation laws for the compressive fluids: energy/matter, momentum, and internal energy. Consequently, we show the new method for computation of the cylindrical expansion (radial expansion w.r.t. the direction of particle propagation) of a massive elementary particle during an inertial propagation in a vacuum with a stable spherically symmetric rest-mass energy density distribution (in particle hydrostatic equilibrium), before an impact with a large and massive obstacle (barrier). This is used to explain also the double-slit experiments with massive particle as, for example, an individual electron. We also propose a practical method to test the QM completion based on these experiments.

Keywords: Quantum Physics, Double-slit experiment, Particle internal dynamics.

1 Introduction

Quantum mechanics, based on the Schrödinger equation is an epistemic statistical theory, here denominated as Statistical Quantum Mechanics (SQM), to differentiate it from the new part of the ontological quantum theory, provided in [6] and [7], denominated Individual particles Quantum Mechanics (IQM). Both of them are necessary components of the quantum theory, as are the Classical Mechanics for Individual objects (ICM),based on the Newton equations, Hamiltonian-Jacobi equations or the Euler-Lagrange equation of motion of individual objects) and the Statistical Classical Mechanics (SCM), based on the Liouville equations.
In the IQM theory there is a deeper specification of the state of the particle, and in this approach to completion provided in [6], these states are specified by the energy-density distributions of a given particle in the Minkowski time-space. Such an ontic state, also not fully accessible (non fully observable by the measurements, and/or with non accessible small compactified higher-dimensions for the electric charge and spin, for example), has to represent the complete description of an individual elementary particle, in order to be able to compute from it all properties of a particle as its rest-mass, position, speed, momentum, total energy, etc...
The standard quantum theory with the probabilistic wavefunctions and their statistical ensemble interpretation is based on the classical concept of a *point-like particle* and do not have the theory able to describe an individual particle with its trajectory and given momentum and energy in any fixed instance of time. Because of that as noted by Einstein it was an incomplete theory, differently from the classic mechanics which has both statistical theory (for example the thermodynamic of a gas) and theory for each individual object (Newton, Euler-Lagrange equations for the motion of an individual object). In the proposed completion of quantum theory [6] instead, an *individual* massive particle's wave-packet (described by the complex scalar wave-packets Ψ in what follows) always occupies a nonzero 3-D volume. It holds also for bosons when they become unstable after an initial 'space explosion' and, consequently, assume the massive particle behavior and a finite but non-zero matter/energy-density volume in open 3-D space. Hence, in this IQM theory [6] for individual elementary particles based on energy-density wave-packets, the point-like particles are only the stable-state bosons when they propagate with speed of light in the vacuum, and with their energy-density distributed in higher compactified dimensions [7]. This new IQM theory for individual particles is able to compute the spectra

of the rest-masses of the particles which is not possible to obtain with the statistical SQM theory. Moreover, it was demonstrated that this quantum theory completion is conservative w.r.t. the theoretical and experimental results of the statistical SQM theory. However, we need also some experimental tests for the validity of this IQM theory for individual particles and here we provide them by double-slit experiments for the massive particles (we recall the fact that, for example, the string theory still did not provide an experimental method for the validation of that IQM theory).

It was shown [3, 4, 5, 6] that, generally, any massive particle can be defined in the Minkowski time-space by the complex wave-packet

$$\Psi = \Phi(t, \overrightarrow{\mathbf{r}})\mathrm{e}^{-i\varphi_T} \tag{1}$$

where $\overrightarrow{\mathbf{r}} = q_1\mathbf{e}_1 + q_2\mathbf{e}_2 + q_3\mathbf{e}_3$ (for the 3-D Minkowski space orthonormal basis vectors $\mathbf{e}_j$, with $\mathbf{e}_j \cdot \mathbf{e}_j = -1$ for $1 \le j \le 3$ and $\mathbf{e}_0 \cdot \mathbf{e}_0 = 1$ for the time-coordinate $q_0 = ct$) composed by two sub components: by the shape $\Phi(t, \overrightarrow{\mathbf{r}})$ of particle's body that is a real function which defines the real *rest-mass energy-density* $\Phi_m \equiv \Psi\overline{\Psi} = \Phi^2(t, \overrightarrow{\mathbf{r}}) \ge 0$, and by the 'phase (pilot) wave' with phase φ_T, $\mathrm{e}^{-i\varphi_T} = \mathrm{e}^{-\frac{i}{\hbar}(\overrightarrow{\mathbf{p}}(\overrightarrow{\mathbf{r}_T}-\overrightarrow{\mathbf{r}_T}(0))+Et)}$, which is a complex function defined only for the particle's barycenter $\overrightarrow{\mathbf{r}_T}(t) \equiv \frac{1}{\mathbf{1}_\Phi}\int \overrightarrow{\mathbf{r}}\Phi_m(t, \overrightarrow{\mathbf{r}})dV$, of the massive elementary particle with the *total energy* E and momentum $\overrightarrow{\mathbf{p}}$ which may change in time as well, and $\mathbf{1}_\Phi \equiv \int \Phi_m(t, \overrightarrow{\mathbf{r}})dV$ is the particle's invariant energy (equal to rest-mass energy m_0c^2 for massive particles and energy E_0 of a boson, measured in the frame in which massive source of this boson is in rest). Thus,

$$m_0 = \int m_0(t, \overrightarrow{\mathbf{r}})dV = \int \frac{\Phi_m(t, \overrightarrow{\mathbf{r}})}{c^2}dV \tag{2}$$

where $m_0(t, \overrightarrow{\mathbf{r}}) \equiv \frac{\Phi_m(t, \overrightarrow{\mathbf{r}})}{c^2}$ is the rest-mass density.

From the fact that a field is a quantity defined at every point $(t, \overrightarrow{\mathbf{r}})$ of the 4-D time-space manifold $\mathcal{M}$, such a quantity can be a complex number of the wave-packet $\Psi = \Phi(t, \overrightarrow{\mathbf{r}})\mathrm{e}^{-i(\overrightarrow{\mathbf{p}}(\overrightarrow{\mathbf{r}_T}-\overrightarrow{\mathbf{r}_T}(0))+Et)/\hbar}$ or a real number of the energy-density $\Phi_m(t, \overrightarrow{\mathbf{r}}) = \Phi^2(t, \overrightarrow{\mathbf{r}}) = \overline{\Psi}\Psi$ (for a massive particle $\curlywedge\Phi_m(t, \overrightarrow{\mathbf{r}})$ is its matter-density, where $\curlywedge$ is the constant which transforms the rest-mass energy into the 'matter').

When a particle propagates in the vacuum with constant speed $\overrightarrow{\mathbf{v}}$ it has the time-invariant distribution $\Phi_m = \frac{K}{\sqrt{r}}$ around its barycenter $\overrightarrow{\mathbf{r}_T}$ [8], corresponding to particle's hydrostatic equilibrium where each infinitesimal amount of particle's material body $\curlywedge\Phi_m(t, \overrightarrow{\mathbf{r}})$ is in rest w.r.t. particle's barycenter.

However, generally, during an acceleration each infinitesimal amount of energy-density $\Phi_m(t, \overrightarrow{\mathbf{r}})$ moves with a different speed $\overrightarrow{\mathbf{w}}(t, \overrightarrow{\mathbf{r}})$ w.r.t. the group velocity $\overrightarrow{\mathbf{v}}(t) = \frac{d}{dt}\overrightarrow{\mathbf{r}}_T(t) = v_1\mathbf{e}_1 + v_2\mathbf{e}_2 + v_3\mathbf{e}_3$, with $v = \| \overrightarrow{\mathbf{v}} \| = \sqrt{v_1^2 + v_2^2 + v_3^2}$ of particle's energy-density wave-packet and it is shown [6] that is satisfied the following relationship

$$\overrightarrow{\mathbf{v}}(t) = \frac{1}{\mathbf{1}_\Phi} \int \overrightarrow{\mathbf{w}}(t, \overrightarrow{\mathbf{r}})\Phi_m(t, \overrightarrow{\mathbf{r}})dV \tag{3}$$

so, we can introduce the variation-velocity of the particle's matter flux $\overrightarrow{\mathbf{u}}(t, \overrightarrow{\mathbf{r}}) = \overrightarrow{\mathbf{w}}(t, \overrightarrow{\mathbf{r}}) - \overrightarrow{\mathbf{v}}(t)$ at each space-time point $(t, \overrightarrow{\mathbf{r}})$ inside particle's matter (where $\Phi_m(t, \overrightarrow{\mathbf{r}}) > 0$). As shown in [6], during an inertial propagation when the particle is in a hydrostatic equilibrium, we have that Φ_m is spherically symmetric around particle's barycenter with $\overrightarrow{\mathbf{u}}(t, \overrightarrow{\mathbf{r}}) = 0$ in every point inside particle's matter, so that every infinitesimal amount of Φ_m propagates with the constant wave-packet group velocity $\overrightarrow{\mathbf{v}}$. Only during the particle's accelerations we have that $\overrightarrow{\mathbf{u}}(t, \overrightarrow{\mathbf{r}}) \neq 0$, so that particle's body changes dynamically its shape in time.

In the assumption [6] of the topology of the matter of an elementary massive particle, the wave-packet do not undergo a spreading, also when it changes its matter density distribution (i.e., its energy-density Φ_m), and tends to its stable stationary spherically symmetric distribution during inertial propagation in the vacuum. That is, the matter has some internal self-gravitational autocohesive force analogously to the peace of *perfect fluid*[1] in the vacuum, so that at any instance of time, the 3-D space topology of particle's matter distribution, and consequently its compressible energy-density Φ_m is simply connected, closed, continuous and differentiable.

Perfect elasticity assumption: Although elasticity is most commonly associated with the mechanics of solid bodies or materials, even the early literature on classical thermodynamics defines and uses "elasticity of a fluid" in ways compatible with the broad definition provided in the introduction above. Throughout the theory of massive elementary particle (in PART I), it is assumed that the particle's bodies undergoing the action of external forces are *perfectly elastic*, i.e., that they resume their initial form (in its hydrostatic equilibrium) com-

[1] We consider that the matter of an particle is a perfect fluid, that is, have no shear stresses, viscosity, or heat conduction. Perfect fluids are often used in general relativity to model idealized distributions of matter, such as the interior of a star or an isotropic universe. In general relativity, a fluid solution is an exact solution of the Einstein field equation in which the gravitational field is produced entirely by the mass, momentum, and stress density of a fluid. In astrophysics, fluid solutions are often employed as stellar models. Consequently, by the assumption that particle's material body is a perfect fluid, we obtain the full physical unification of the QM with universe.

pletely after removal of forces.
The elastic body of an elementary particle has no any molecular internal structure and hence the particle's matter is *homogeneous* and continuously distributed over its volume so that the smallest element cut from the body possesses the same specific physical properties as the whole body. That is, it is assumed that the particle's body is *isotropic*, i.e., that the elastic properties are the same in all spatial directions. Moreover, we have no any thermodynamic dissipation inside particle's body during its elastic deformations, that is, we have no thermal losses of particle's internal energy during such elastic deformations caused by the external forces: the particle's kinetic energy converted into its internal elastic potential energy V during elastic deformations have no any side-effects of thermal losses.
The essence of elasticity is the reversibility. Forces applied to an elastic material transfer energy into the material which, upon yielding that energy to its surroundings, can recover its original shape. Elastic energy occurs when objects are compressed and stretched, or generally deformed in any manner. For a massive elementary particle it corresponds to energy stored by changing the internal forces of particle's hydrostatic equilibrium based on particle's self-gravitational force.
Such properties of the particle's body differentiate it from the structural materials composed by the molecular structures as all material object used in our every-day practice. Each massive elementary particle satisfies the following conservation laws:
1. Conservation of the matter/energy law: Analogously to the Euler first equation of fluid dynamics (continuity equation), which represents the conservation of mass, here we have the analog equation for the conservation of matter (that is of the particle's rest-mass energy):

$$\frac{\partial \Phi_m(t, \overrightarrow{\mathbf{r}})}{\partial t} + \nabla \cdot (\Phi_m(t, \overrightarrow{\mathbf{r}}) \overrightarrow{\mathbf{w}}(t, \overrightarrow{\mathbf{r}})) = 0 \tag{4}$$

In what follows, for the Cartesian coordinate system, $\nabla = \mathbf{e}_1 \frac{\partial}{\partial x} + \mathbf{e}_2 \frac{\partial}{\partial y} + \mathbf{e}_3 \frac{\partial}{\partial z}$ is the gradient (for $x \equiv q_1, y \equiv q_2$ and $z \equiv q_3$) so that the Laplacian is defined by $\triangle = -\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ (we are using positive-time metric signature (+,-,-,-))
In Section 2.7 in [6], dedicated to the 3-D radial expansion of the bosons w.r.t. the direction of particle's propagation, to the tunneling and reflections, has been considered the cylindrical expansion of the massive boson with energy density Φ_m (that is, during the unstable boson's states where the variation-velocity

$\overrightarrow{\mathbf{u}}(t, \overrightarrow{\mathbf{r}})) \neq 0$.
Here, instead, we will consider the same radial (i.e., cylindrical) extension for the massive particles, and to explain why in the the double-slit experiments for the massive particle's, described in [6] (Section 4.6.1) this particle's radial "explosion" in from of the large massive barrier transforms, in an infinitesimally short interval of time, its spherically symmetric micro body-density into a very large macro-disk able to cover both slits contemporarily and to permit the propagation of the particle's body density trough both slits contemporarily.
It was explained [8] why in the stable particle's states, during an inertial propagation with the constant speed $\overrightarrow{\mathbf{v}}$ in the vacuum (sufficiently far from another particles), we have no internal motion of the rest-mass density of the particle, that is, we have that $\overrightarrow{\mathbf{u}}(t, \overrightarrow{\mathbf{r}})) = 0$ in each point inside particles rest-mass density distributions (where $\overrightarrow{\mathbf{r}}$ is the vector from the barycenter to the observed point).
We need that the body of the particle $\leftthreetimes\Phi_m$ provides also the physical internal pressure $P(t, \overrightarrow{\mathbf{r}})$ (which is a non-geometrical property) in order to guarantee the hydrostatic equilibrium of the massive particles. The hydrostatic equilibrium of an massive elementary particle demonstrated that the body of this particle $\leftthreetimes\Phi_m$ is a material substance [8], which is fluid and elastic, and which can not be reduced to the time-space geometry[2]. So, we will use other two conservation laws for the elementary particles, provided in [8]:
2. Conservation of the momentum law: we use the following Cauchy momentum equation (in convective form derived from the second Newton's law) in a time-space point $(t, \overrightarrow{\mathbf{r}})$ inside particle's body where $\Phi_m(t, \overrightarrow{\mathbf{r}}) \neq 0$,

$$\frac{d\overrightarrow{\mathbf{u}}}{dt} = \frac{c^2}{\Phi_m}\nabla \cdot \tau - \frac{c^2}{\Phi_m}\nabla P + \overrightarrow{\mathbf{g}} \tag{5}$$

where τ in the 2-tensor which express the viscosity properties. In what follows we will consider that there is no any significant viscosity inside particle's body (a perfect fluid), so that $\nabla \cdot \tau = 0$, and hence the equation above is equal to the Euler momentum equation (otherwise we would obtain the Navier-Stokes momentum equation).
Here we consider that $\overrightarrow{\mathbf{g}}$ is equal to the particle's self-gravitational internal force (by considering that locally it is much bigger than the external gravitational force). Obviously, the stress terms on the right side of the equation above are yet unknown (they are the hidden variables of an elementary particle which can not

[2]This real physical material substance generates a curved time-space curvature inside and around it (the micro-island curvature), but this material substance can not be simply 'generated by time-space curvature'. That is, a massive elementary particle can not be reduced to the pure geometry

be measured in every point $(t, \overrightarrow{\mathbf{r}})$ inside particle's body where $\Phi_m(t, \overrightarrow{\mathbf{r}}) \neq 0$), so that this equation can not be used to solve the dynamic phenomena of the perturbations of particle's hydrostatic equilibrium. However, they are useful in order to understand the internal dynamics of the particle's rest-mass energy flow during the accelerations of the particle (when particle is not more in the hydrostatic equilibrium because the vaiation-velocity $\overrightarrow{\mathbf{u}}(t, \overrightarrow{\mathbf{r}})$ of the flow (in the coordinate system with the center in the particle's barycenter) in equation above is not more zero and depends on time as well.

3. Conservation of internal energy law:

$$\frac{de}{dt} = -\frac{c^2 P}{\Phi_m} \nabla \cdot \overrightarrow{\mathbf{u}} \tag{6}$$

which is the 'third Euler's equation' (obtained a century later).

Note that the specific internal energy e keeps account of the gains and losses of energy of the system that are due to changes in its internal state. The internal energy of a given state of a massive elementary particle cannot be directly measured.

However, it is fundamental concept which explains the principles of 'internal energy V', introduced [6] (in Definition 5 for internal dynamic assumption) and described in Section 2.6 in [6], dedicated to the phenomena of 'virtual particles' (which does not satisfy the energy relationship $E^2 = m_0^2 c^4 + c^2 p^2$ for the massive particle with the total energy E and momentum p), to the phenomena of massive bosons and to the physical explanation of Higgs mechanism. In all these more complex internal dynamic phenomena of the massive particles, it is valid the energy equation $(E + V)^2 = m_0^2 c^4 + c^2 p^2$, where E is the measurable total energy of the particle (which during 3-D space breaking of an inertial propagation of the particle (considered as a closed system) can *remain constant*; but this breaking of 3-D symmetry produces the changing of particle's body shape [6]). V is a potential internal energy (based on this specific internal energy e in the equation above) used, for example, during the spatial expansion of the particle's body (during strong excitations) when has to be spent some energy against the autocohesive self-gravitational force inside particle's body which dynamically changes particle's shape (with the density flow velocity $\overrightarrow{\mathbf{u}}$) and its internal pressure P.

In next section, we provide an ideal model of cylindrical expansion of a massive elementary particle. Then, in Section 3, we provide the dynamics of this radial explosion which generates the large radius of a disc-shape of particle's body in an infinitesimal interval of time able to cover both slits contemporarily in the double-slit experiments. The application of this radial explosion of the massive particles to the double-slit experiments is presented in the last section, and is provided a method for experimental test of this phenomena and hence of the completion (for the individual particles) of the quantum theory by the IQM.

2 A model of cylindrical expansion of an elementary particle

Let us consider the case of the double-slit experiments with the relatively low-energy electrons, and not the high-energy electrons which would be absorbed by the barrier. In fact, the high-energy electrons have very high kinetic energy, that is, the velocity of propagation, so that the impact with the large massive boundary would be to much fast in order to have enough time for the significant radial expansion before electron's impact with barrier. Thus, such a very high-energy electron will have impact only with one massive particle in the barrier and hence only two possibilities: if electron's energy is not too much high then the Compton effect inside the barrier would happen and this electron, after a number of collisions with the particles in this massive barrier, will lose its kinetic energy and remain inside the barrier; otherwise, with too much high energy, the collision with another particle inside the barrier would produce the fusion (absorption) with this particle and possible generation of another new particles.

Thus, we consider only the low-energy electrons that propagate with relatively small velocity toward the barrier. So, when this particle becomes very close to this barrier then the process of the radial expansion begins, because its previous inertial propagation in the condition of the perfect 3-D space vacuum symmetry now is interrupted by the sharp breaking of this 3-D space symmetry, caused by the large massive barrier in front of it. Thus, we have that the boundary conditions for this electron (for the differential equation (4) of its matter/energy conservation law) and its local micro-island curvature around electron's body, generated by its self-gravitational force, are now drastically changed.

From the fact that the speed of this electron toward the barrier is not enormously high, the interval of time, from the beginning of particle's radial expansion and the physical impact (of such a frontally-enlarged electron) with the massive particles in the barrier, would be sufficiently long for particle's radial expansion. Hence, the enlarged frontal surface of the electron, which will have during the physical impact with the barrier, will cover an enormous number of massive particles in the barrier's surface (if the radial expansion of electron's frontal surface is extremely fast - what we have to verify in next). Notice that such a low-energy electron will also preserve the elastic properties during the interactions with the barrier (i.e., the elastic compression of its matter-density) during the frontal collision with the particles in the barrier, which changes its internal potential energy V like in the Hooke's law (see at [6], page 108, after Example 11). From the fact that this radial (w.r.t. the direction of propagation) disk of the enlarged electron, parallel with the barrier, will have a physical impact with the thousands of particles belonging to the external surface of the barrier, we will not have the Compton effect, but only a distribution of particle's impact over a large surface of the barrier (in which the impulse-impact with any single particle would be relatively small). Hence, the electron's body will only continue its radial expansion over the external surface of the barrier, by transferring its initial kinetic energy into a (negative) internal potential energy V (the decrement of the positive kinetic part of energy E is represented by decrement of $E+V$, that is, by increment of $|V|$ where V is negative, so that V effectively decreases; this decrease of kinetic energy explains the decrease of particle's speed) which would change its internal gradient-pressure force $\nabla P(t, \overrightarrow{\mathbf{r}})$ and will be used against electron's internal self-gravitational force $\frac{\Phi_m(t,\overrightarrow{\mathbf{r}})}{c^2}\overrightarrow{\mathbf{g}}(t, \overrightarrow{\mathbf{r}})$, by producing a great radial speed of expansion (from the momentum conservation law (5)) $\overrightarrow{\mathbf{u}}(t, \overrightarrow{\mathbf{r}})$, such that

$$\frac{d\overrightarrow{\mathbf{u}}(t, \overrightarrow{\mathbf{r}})}{dt} = -\frac{c^2}{\Phi_m(t, \overrightarrow{\mathbf{r}})}\nabla P(t, \overrightarrow{\mathbf{r}}) + \overrightarrow{\mathbf{g}}(t, \overrightarrow{\mathbf{r}}) \tag{7}$$

up to the moment when particle's disc-body would find in the barrier some slit to pass throw it, or up to the total annulation of its kinetic energy. In the second case, the electron will be arrested on the surface of the barrier and then the internal negative energy V will be reversed now into the positive kinetic energy (hence now $E+V$ will be incremented, by decrement of $|V|$ (of the negative V and hence V effectively increases); the increase of kinetic energy explains the increase of particle's velocity), and hence will begin the process of (perfect) reflection of this electron from the barrier. In the moment when all internal potential energy is transformed into particle's kinetic energy, we will have again a stable electron (in its hydrostatic equilibrium) which now propagates in the opposite direction with the same constant speed as the speed before the impact with the barrier.

Let us consider an inertial propagation of a massive particle along the axis x, toward a large massive barrier up to the time instance $t=0$, during which it is in the hydrostatic equilibrium with a spherically symmetric energy density $\Phi_m = \frac{K}{\sqrt{\rho}}$ [8], where ρ is the distance from particle's barycenter, with the positive constant $K = \frac{2.5 m_0 c^2}{4\pi r_0^{5/2}}$ and the radius of particle's spherical body r_0. We assume that the center of the cylindrical coordinate system, with coordinates $\overrightarrow{\mathbf{r}} \equiv (r, \theta, x)$, is in particle's barycenter. Then, we consider that, for $t > 0$, this particle is influenced by the breaking of the 3-D spatial symmetry, caused by this massive barrier. So, we have that in this cylindrical coordinate system $\rho = (r^2 + x^2)^{1/2} \leq r_0$ and hence $\Phi_m(t, r, x) = K(r^2 + x^2)^{-1/2}$ is a spherically symmetric for all $t \leq 0$. That is, $\Phi_m(0, r, x) \neq 0$ for $(r^2 + x^2)^{1/2} \leq r_0$ represents the initial hydrostatic equilibrium of the particle's material body with the radius r_0 and velocity $\overrightarrow{\mathbf{u}} = 0$ in all point inside particle's body.

Thus, let us consider the solution of the differential equation (4) for the rest-mass energy density Φ_m for $t > 0$, by considering that in this coordinate system the group velocity $\overrightarrow{\mathbf{v}}(t) = 0$ so that $\overrightarrow{\mathbf{w}}(t, \overrightarrow{\mathbf{r}}) = \overrightarrow{\mathbf{v}}(t) + \overrightarrow{\mathbf{u}}(t, \overrightarrow{\mathbf{r}}) = \overrightarrow{\mathbf{u}}(t, \overrightarrow{\mathbf{r}}) = u_r \mathbf{e}_r + u_\theta \mathbf{e}_\theta + u_x \mathbf{e}_1$.

In this simple idealized model, we consider the case when $|\overrightarrow{\mathbf{u}}(t, \overrightarrow{\mathbf{r}})| > 0$ only for $\rho = (r^2 + x^2)^{1/2} \geq r_m$ where r_m is an infinitesimal radius inside the body in which this perturbation does not change the density during particle's expansion, i.e., $0 \approx r_m << r_0$. So, the equation (4), rewritten in these cylindrical coordinates (r, θ, x), becomes equal to

$$-\frac{\partial \Phi_m}{\partial t} = \nabla \cdot (\Phi_m \overrightarrow{\mathbf{w}}) = \frac{1}{r}\left(\frac{\partial}{\partial r}(r u_r \Phi_m) + \frac{\partial (u_\theta \Phi_m)}{\partial \theta} + r \frac{\partial (u_x \Phi_m)}{\partial x}\right) \tag{8}$$

which, in this case of the radial expansion with $u_\theta = u_x = 0$ and $u_r = u_r(t, r, x)$, reduces to the simple equation[3]

$$-\frac{\partial \Phi_m(t, r)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(r u_r \Phi_m) = \frac{1}{r}\left(u_r \Phi_m + r \frac{\partial u_r}{\partial r}\Phi_m + r u_r \frac{\partial \Phi_m}{\partial r}\right) \tag{9}$$

considered for $r \geq r_m > 0$ and $t > 0$ (because otherwise both sides of this equation are trivially equal to zero).

Let us seek a solution, for such a radial expansion of the particle's body when $t \geq 0$, by separating the variables. However, we can not use the complete separation of independent variables $\Phi_m(t, x, r) = T(t)R(r)X(x)$ because we

[3] Notice that in the observer's reference Minkowski time-space coordinate system (of the quantum laboratory), for a given angle θ of the cylindrical coordinates, we have that for $\overrightarrow{\mathbf{r}} = x\mathbf{e}_1 + r\cos\theta\mathbf{e}_2 + r\sin\theta\mathbf{e}_3$, $\overrightarrow{\mathbf{u}}(t, \overrightarrow{\mathbf{r}}) = 0\mathbf{e}_1 + u_r(t, x, r)\cos\theta\mathbf{e}_2 + u_r(t, x, r)\sin\theta\mathbf{e}_3$ where $\mathbf{e}_1 = \mathbf{e}_x$ is the unit vector in the direction x of particle's propagation, while the absolute speed is $\overrightarrow{\mathbf{w}}(t, \overrightarrow{\mathbf{r}}) = \overrightarrow{\mathbf{v}}(t) + \overrightarrow{\mathbf{u}}(t, \overrightarrow{\mathbf{r}}) = v(t)\mathbf{e}_1 + u_r(t, x, r)\cos\theta\mathbf{e}_2 + u_r(t, x, r)\sin\theta\mathbf{e}_3$, where $v(t)$ is the particle's velocity toward the barrier at the time instance $t \geq 0$. Such a Minkowski time-space coordinate system is used for the definition of the radial-expansion operators T_x [6](in Proposition 11, Section 2.8, where for u_r is used the symbol v_r).

are looking for the particle's expansion from its hydrostatic equilibrium when $\Phi_m(0, x, r) = \frac{K}{\sqrt{r^2+x^2}}$ where is impossible to separate the variables r and x. Hence, we can only use the following separation $\Phi_m(t, x, r) = T(t)R(x, r)$, where $T(t)$ is a dimensionless function while $R(x, r)$ is a 3-D energy density function $[Joule/cm^3]$ and $u_r(t, x, r) = u_T(t)u_R(x, r)$ where $u_T(t) \geq 0$ is a function in $[sec^{-1}]$ and $u_R(x, r)$ is a 1-D function $[cm]$(which express a distance). The function $T(t)$ is the time evolution of the rest-mass energy density and hence we fix its initial value at $t = 0$ by $T(0) = 1$. We fix also $u_T(0) = 0$, with $u_T(t) > 0$ for $0 < t < \triangle t$ where $\triangle t$ is the short time-interval during which an 3-D symmetry breaking generates this perturbation which transforms the hydrostatic-equilibrium sphere with the radius r_0 into a growing (also macroscopical) disk but microscopically wide (the thickness of this disk is $2r_0$, with $-r_0 \leq x \leq r_0$).

Notice that $u_T(t)$ *is not* the time evolution of the radial velocity, and also $u_R(x, r)$ is not a velocity; only their product represents the radial velocity. Thus, at $t = 0$ we have still the hydrostatic equilibrium with $\Phi_m(0, x, r) = T(0)R(x, r) = R(x, r) = \frac{K}{\sqrt{r^2+x^2}}$ if $r^2 + x^2 \leq r_0^2$ (0 otherwise), and the energy density velocity $u_r(0, x, r) = u_T(0)u_R(x, r) = 0$ (because, from above, $u_T(0) = 0$). Consequently, the equation (9) reduces into the following differential equation, for $r > r_m > 0$ and $t > 0$ (note that differently from $\Phi_m(0, x, r)$, for $R(x, r)$ we allow any finite big value for r, with $|x| \leq r_0$),

$$\frac{1}{u_T(t)T(t)}\frac{\partial T}{\partial t} = k = -\frac{u_R(x, r)}{R(x, r)}\frac{\partial R}{\partial r} - \frac{1}{r}u_R(x, r) - \frac{\partial u_R}{\partial r} \tag{10}$$

where k is a *dimensionless* constant real value (because left-hand side depends only on the free time-variable t and right-hand side depends only on the free space-variables x and r (radial coordinate). All functions different from $u_R(x, r)$ are positive, while the radial velocity component $u_R(x, r) \geq 0$ for the radial *expansion*, and $u_R(x, r) \leq 0$ for the radial *compression* of the particle's energy-density distribution inside a disk of time-dependent radius $r_0(t, x)$ during particle's de/accelerations. We recall that, after radial explosion, in the case of the perfect reflection from the large massive barrier, this large disk returns to the spherically symmetric micro sphere with radius r_0 of particle's hydrostatic equilibrium and hence, after that, this particle again propagates with the constant speed but in the opposite direction.

We consider ideally infinite boundary of the barrier external surface, orthogonal to the direction of the particle's propagation. We will see how the enlargement of this disk is different for different values of $-r_0 \leq x \leq r_0$. In what follows,

we consider that the radial velocity $u_r(t,x,r) = u_T(t)u_R(x,r)$ is determined by the particles speed toward the barrier and by the dimension of the barrier which causes this 3-D symmetry breaking perturbation, so that other dynamic variables can be expressed by using $u_T(t)$ and $u_R(x,r)$.
From the left-hand side of the equation above, we obtain the following solution for the time-dependent component of the energy-density distribution $T(t) > 0$ for any $t \geq 0$:

$$T(t) = \mathrm{e}^{k\int_0^t u_T(s)ds} \tag{11}$$

Thus, from the fact that all functions at the right-hand side of this equation are positive and $t \geq 0$, in order to obtain that the $T(t)$ component of the energy density diminishes with time during particle's expansions (because the total rest-mass energy is invariant), we conclude that:

- During the radial expansion, when $u_R(x,r) \geq 0$, we must have that $k < 0$. The energy density diminishes with time in any point inside the particle's body with the disk radius $r_0(t,x)$ that increases with time;
- During the radial compression, when $u_R(x,r) \leq 0$, we must have that $k > 0$. The energy density increases with time in any point inside the particle's disk body with radius $r_0(t,x)$ which decreases with time.

Remark: the value $|k|$ is proportional to the force of the impact of the particle with this barrier which causes this radial-expansion perturbation from the particle's hydrostatic equilibrium (micro-sphere with an extremely small radius r_0), i.e., proportional to the particle's speed. With a greater speed we have a greater kinetic particle's energy, which can be transformed into a negative internal potential energy V used for radial expansion against particle's self-gravitational force, so that the radial speed of particles radial expansion would be greater. In fact, a greater value of $|k|$ would produce stronger changes of $T(t)$ (which represents the time-evolution of the particle's rest-mass matter/energy density), that is, would produce a faster expansion of the particle's disk and hence its frontal boundary would cover a greater surface of the barrier in a smaller interval of time. Consequently, k is a fundamental real parameter, which represents the strength of this impact-force caused by the 3-D symmetry breaking during an inertial propagation of the particle, and will be considered in what follows as a given (non derived) parameter in all equations.
$\square$
From the right-hand side of equation (10), we obtain the following dependence

of the energy-density distribution component $R(x,r) \geq 0$ on the radial velocity component $u_R(x,r)$, for $0 \approx r_m \leq r$ (for which $u_R(x,r) \neq 0$) :

$$\frac{1}{R(x,r)}\frac{\partial R(r)}{\partial r} = -(\frac{k}{u_R(x,r)} + \frac{1}{r} + \frac{1}{u_R(x,r)}\frac{\partial u_R(x,r)}{\partial r}) \tag{12}$$

Notice that for $0 \leq r < r_m \approx 0$, the rest-mass energy density $\Phi_m(t,x,r)$ does not change in time and remains equal to that in the hydrostatic equilibrium. The radial component of the density velocity $u_R(x,r)$, for $r_m \leq r$, depends on the rest-mass density $\Phi_m(0,r)$ of the particle in the hydrostatic equilibrium as follows (by replacing $R(x,r)$ by $\Phi_m(0,x,r) = \frac{K}{\sqrt{\rho}}$ in (12), where $\rho = \sqrt{r^2+x^2} \leq r_0$ is the distance from particle's barycenter):

$$\frac{\partial u_R}{\partial r} = -k - u_R(x,r)(\frac{1}{r} + \frac{1}{\Phi_m(0,x,r)}\frac{\partial \Phi_m(0,x,r)}{\partial r}) = -k + u_R(x,r)\frac{r^2 - 2\rho^2}{2r\rho^2} \tag{13}$$

It is easy to verify that the solution of this differential equation (13) is given by

$$u_R(x,r) = -\frac{2}{3}k\frac{\rho^2}{r} = -\frac{2k}{3}\frac{r^2+x^2}{r} \tag{14}$$

so that we have the following general result:

Proposition 1 *From the fact that* $|\frac{1}{u_R(x,r)}|$ *is a finite value for all* $r \geq r_m$ *and* $-r_0 \leq x \leq r_0$*, the general solution of differential equation (12), compatible with the initial hydrostatic equilibrium of the massive particle, for* $r \geq r_m$*, is*

$$R(x,r) = \frac{k_2}{r_0}\frac{1}{u_R(x,r)r}\mathrm{e}^{-k\int_{r_m^2-x^2}^{r^2}(2\sqrt{s}\ u_R(x,\sqrt{s}))^{-1}ds} \tag{15}$$

where $k_2 \neq 0$ *is a (positive or negative) real constant (in* $[Joule]$*) such that* $\frac{k_2}{u_R(x,r)} > 0$*, given by*

$$k_2 = -\frac{5k}{3}\frac{m_0c^2}{4\pi}(\frac{r_m}{r_0})^{3/2} \tag{16}$$

where $0 \approx r_m << r_0$*.*

Proof: It is easy to verify that (15) is the solution of the differential equation (12) where for each fixed value of x (considered not as variable but as a constant parameter) partial derivatives can be substituted by total (material) derivatives, and hence by Leibniz integral rule we have that hplds
$\frac{d}{dr}(-k\int_{r_m^2-x^2}^{r^2}(2\sqrt{s}\ u_R(x,\sqrt{s}))^{-1}ds) = -k(2\sqrt{s}\ u_R(x,\sqrt{s}))^{-1}|_{s=r^2}\frac{d}{dr}r^2$
$= -k(2\sqrt{r^2}\ u_R(x,r))^{-1}2r = -\frac{k}{u_R(x,r)}$,

because the function $\frac{1}{2\sqrt{s}\;u_R(x,\sqrt{s})}$ in the integral is finite in the range of the integration. In effect, from (14), we have that $\frac{1}{2\sqrt{s}\;u_R(x,\sqrt{s})} = -\frac{3}{4k}\frac{1}{s+x^2}$, which for the interval of integration $[r_m^2 - x^2, r^2]$ changes from $-\frac{3}{4k}\frac{1}{r_m^2}$ to $-\frac{3}{4k}\frac{1}{r^2+x^2}$, and hence is always finite and integrable in this interval.
So, we have that
$\frac{\partial}{\partial r}\mathrm{e}^{-k\int_{r_m^2-x^2}^{r^2}(2\sqrt{s}\;u_R(x,\sqrt{s}))^{-1}ds} = -\frac{k}{u_R(x,r)}\mathrm{e}^{-k\int_{r_m^2-x^2}^{r^2}(2\sqrt{s}\;u_R(x,\sqrt{s}))^{-1}ds}$
and hence it is easy to verify that $R(x,r)$ given by equation (15) satisfies the differential equation (12).

Now, from (14), we obtain that $\mathrm{e}^{-k\int_{r_m^2-x^2}^{r^2}(2\sqrt{s}\;u_R(x,\sqrt{s}))^{-1}ds} = \mathrm{e}^{\frac{3}{4}\int_{r_m^2-x^2}^{r^2}(s+x^2)^{-1}ds} = \mathrm{e}^{\frac{3}{4}ln((r^2+x^2)/r_m^2)} = (r^2+x^2)/r_m^2)^{3/4} = (\rho)/r_m)^{3/2}$, and hence
$R(x,r) = \frac{k_2}{r_0}\frac{1}{u_R(x,r)r}\mathrm{e}^{-k\int_{r_m^2-x^2}^{r^2}(2\sqrt{s}\;u_R(x,\sqrt{s}))^{-1}ds} = \frac{k_2}{r_0}\frac{1}{u_R(x,r)r}(\rho)/r_m)^{3/2} =$
$-\frac{3k_2}{2kr_0\rho^2}(\rho^2)/r_m^2)^{3/4} = -\frac{3k_2}{2kr_0r_m^{3/2}}\frac{1}{\rho^{1/2}}$, and from the fact that from the hydrostatic equilibrium we have that also $R(x,r) = \Phi_m(0,x,r) = \frac{K}{\rho^{1/2}}$, so that $K = -\frac{3k_2}{2kr_0r_m^{3/2}}$ and hence from $K = \frac{2.5m_0c^2}{4\pi r_0^{5/2}}$ [8] we obtain the equation (16).
□

Remark: Notice that the solution is $R(\rho) = \frac{K}{\rho^{1/2}}$, where $\rho = \sqrt{r^2+x^2}$ is the distance from the particle's barycenter, so that $R(\rho)$ is equal to the solution obtained for small spherical particle's perturbations studied for the particle's self-stability in [8].
From (13) we obtain that $u_R(x,r) = -\frac{2k}{3}\frac{\rho^2}{r}$, for a given 3-D symmetry breaking perturbation with the strength fixed by the value of k. The fact that the radial velocity component $u_R(x,r)$ increments with $\frac{\rho^2}{r} = r + \frac{x^2}{r}$ is compatible with the necessities of particle's "radial explosions" in extremely excitations during extremely short time-intervals required by the IQM theory in [6]. It will be analyzed in details in next section.
□

3 Dynamics of the cylindrical explosion

Let us fix at $t = 0$ the beginning of the radial explosion of the particle, from its spherical form with radius r_0 of its hydrostatic equilibrium when for $\rho = \sqrt{r^2+x^2} \leq r_0$, $\Phi_m = \frac{K}{\sqrt{\rho}}$, where $K = \frac{2.5m_0c^2}{4\pi r_0^{5/2}}$ [8]. From the fact that we have the extension only in the radial direction (w.r.t. to the particle's direction of propagation), this dynamics will transform a micro-spherical particle's body

into a kind of a disk with an irregular border, because the expansion velocity varies with x for each $-r_0 \le x \le r_0$, so that the border surface of this cylindric form is not flat as in a standard disk.
Let us consider now these dynamic changes of this hydrostatic equilibrium density during the strong 3-D symmetry breaking perturbations:

Proposition 2 *Let $r_0(t,x)$ be the radial expansion of the particle's disk, for a given x such that $-r_0 \le x \le r_0$ and time-instance $t \ge 0$, with $r_0(0,x) = \sqrt{r_0^2 - x^2}$ and hence with the radius of the particle's sphere in its hydrostatic equilibrium $r_0(0,0) = r_0$. Then, the dynamic components of the particle's density and radial velocity time-changing are given by:*

$$T(t) = (\frac{r_0^2}{r_0^2(t,x)+x^2})^{3/4} \le 1, \quad and \qquad u_T(t) = -\frac{3}{2k}\frac{r_0(t,x)}{r_0^2(t,x)+x^2}\frac{dr_0(t,x)}{dt} \ge 0 \tag{17}$$

The changing of the particle's rest-mass energy density during a small spherical perturbations for $t > 0$ and $r_m \le r \le r_0(t)$ is given for each fixed x, $-r_0 \le x \le r_0$, and $r \le r_0(t)$, by

$$\Phi_m(t,x,r) = \frac{2.5 m_0 c^2}{4\pi r_0}\frac{1}{r_0^{3/2}(t,0)}\frac{1}{(r^2+x^2)^{1/4}} \tag{18}$$

Proof: We obtain the second equation in (17) from the fact that the radial velocity is defined by $\frac{dr_0(t,x)}{dt} = u_r(t,x,r_0(t,x)) = u_T(t)u_R(x,r_0(t,x))$ and from (14), we obtain

$$\frac{dr_0(t,x)}{dt} = -\frac{2k}{3}\frac{r_0^2(t,x)+x^2}{r_0(t,x)}u_T(t) \tag{19}$$

so, from this equation we obtain the second equation in (17).
Now, from (11) and $u_T(t)$, rewritten in the form for any fixed x (which does not change in time and hence we can use the material derivation $\frac{d}{dt}$), as
$u_T(t) = \frac{3}{4k}\frac{d}{dt}ln(\frac{r_0^2}{r_0^2(t,x)+x^2})$,
we can compute the time-evolution function $T(t)$:
$T(t) = \mathrm{e}^{k\int_0^t u_T(s)ds} = \mathrm{e}^{\frac{3}{4}\int_0^t \frac{d}{ds}(ln(\frac{r_0^2}{r_0^2(s,x)+x^2}))ds} = \mathrm{e}^{-\frac{3}{4}ln(\frac{r_0^2}{r_0^2(t,x)+x^2})}$
$= (r_0^2/r_0^2(t,x)+x^2)^{3/4}$.
From the definition $\Phi_m(t,x,r) = T(t)R(x,r) = (r_0/r_0(t,0))^{3/2}\frac{K}{\sqrt{\rho}}$ (from the fact that $T(t)$ does not depend on x we set $x=0$ so that $T(t)|_{x=0} = (r_0/r_0(t,0))^{3/2}$) and hence from $K = \frac{2.5m_0c^2}{4\pi r_0^{5/2}}$, we obtain the equation (18).
□

It is easy to verify that (18), for $t = 0$, reduces to the hydrostatic equilibrium $\Phi_m(0, x, r) = \frac{K}{\sqrt{\rho}} = \frac{K}{\sqrt[4]{\sqrt{r^2+x^2}}}$.
All equations obtained in the proposition above, expressed by the value of the expansion radius of the particle's disk $r_0(t, x)$, are obtained from the matter/enetgy conservation law (continuity equation), and we need another equation to compute $r_0(t, x)$. Hence, for the computation of the time evolution of the particle's body radius $r_0(t, x)$ during its radial expansion/compression we have to use the momentum conservation law.
The particle's dynamics during the 3-D symmetry breaking, caused by the impact with a large massive barrier, represents the changing of the internal force inside particle's body generated by the gradient of the internal pressure,

$$\nabla P(t, x, r) = \frac{\partial P}{\partial r}\mathbf{e}_r + \frac{\partial P}{\partial x}\mathbf{e}_1 \tag{20}$$

and by considering the self-gravitational acceleration $\overrightarrow{\mathbf{g}}(t, x, r) = g_r(t, x, r)\mathbf{e}_r + g_x(t, x, r)\mathbf{e}_1$, from the momentum conservation law (5) with $\nabla \cdot \tau = 0$ (without viscosity properties of the particle's body), we obtain that

$$\frac{d\overrightarrow{\mathbf{u}}}{dt} = \frac{du_r(t, x, r)}{dt}\mathbf{e}_r = -\frac{c^2}{\Phi_m(t, x, r)}\nabla P(t, x, r) + \overrightarrow{\mathbf{g}}(t, x, r) \tag{21}$$

and hence the following results:

Lemma 1 *In the absence of the viscosity, during the 3-D symmetry breaking perturbations, also the internal pressure $P(t, x, r)$ is a cylindrically symmetric with its gradient inside particle's body, for $t \geq 0$, $r \leq r_0(t, x)$ and $|x| \leq r_0$,*

$$-\frac{\partial P(t, x, r)}{\partial r} = T(t)\frac{K}{c^2\sqrt{\rho}}[-\frac{2k}{3}\frac{\rho^2}{r}(\frac{du_T}{dt} - \frac{2k}{3}(1 - \frac{x^2}{r^2})u_T^2(t)) + \int_{-r_0}^{r_0} T(t)\frac{KG}{c^2}f(t, x, r, s)ds] \tag{22}$$

where $r_0(t, x)$ is the radius of particle's disk-body in the time instance t in the plane defined by x, and $\rho = \sqrt{r^2 + x^2}$. The expression inside the integral represents the contribution of the energy-density in the plane $x = s$ of the particle's disk to the radial component of the internal self-gravitational force with

$$f(t, x, r, s) = \int_0^{r_0(t,s)} dr' \frac{r'}{(r'^2 + s^2)^{1/4}} \int_0^{2\pi} \frac{(r'\cos\theta - r)d\theta}{(x^2 + r^2 + s^2 + r'^2 - 2xs - 2rr'\cos\theta)^{3/2}} \tag{23}$$

Proof: The equation (21) can be reduced into $\frac{d\overrightarrow{\mathbf{u}}}{dt} = \frac{du_r(t,x,r)}{dt}\mathbf{e}_r = -\frac{c^2}{\Phi_m(t,x,r)}\frac{\partial P(t,x,r)}{\partial r}\mathbf{e}_r + g_r(t, x, r)\mathbf{e}_r$ because, in this simplified model, we have only the radial component of the flow speed vector $\overrightarrow{\mathbf{u}}(t,x,r)$ and hence the hydrostatic equilibrium in the x direction, $-\frac{c^2}{\Phi_m}\frac{\partial P}{\partial x}\mathbf{e}_1 + g_x(t,x,r)\mathbf{e}_1 = 0$, and by the fact that $g_r(t,x,r) = -\overrightarrow{\mathbf{g}}\mathbf{e}_r$ (from our positive-time Minkowski metrics where for the spatial unit vectors we have that $\mathbf{e}_j\mathbf{e}_j = -1$ for $j = 1,2,3$ and radial unit vector $\mathbf{e}_r$ can be expressed as a linear expression of them), we obtain

$$\frac{du_r(t,x,r)}{dt} = -\frac{c^2}{\Phi_m(t,x,r)}\frac{\partial P(t,x,r)}{\partial r} - \overrightarrow{\mathbf{g}}(t,x,r)\mathbf{e}_r \tag{24}$$

Thus, from this equation and for $\frac{\Phi_m}{c^2} = \frac{1}{c^2}T(t)\frac{K}{\sqrt{\rho}}$, we obtain that
$-\frac{\partial P}{\partial r} = \frac{\Phi_m}{c^2}(u_R(x,r)\frac{du_T}{dt} + u_R(x,r)\frac{\partial u_R}{\partial r}u_T^2(t) + \overrightarrow{\mathbf{g}}\mathbf{e}_r) = T(t)\frac{K}{c^2\sqrt{\rho}}(-\frac{2k}{3}\frac{\rho^2}{r}(\frac{du_T}{dt} - \frac{2k}{3}(1 - \frac{x^2}{r^2})u_T^2(t)) + \overrightarrow{\mathbf{g}}\mathbf{e}_r)$, where for $\overrightarrow{\mathbf{r}} = (r,0,x)$ (we can fix the angle to 0 because, from the cylindrical symmetry, we obtain the same value independently on the angle) and $\overrightarrow{\mathbf{r}}' = (r',\theta,s)$ we have that
$\overrightarrow{\mathbf{g}}\mathbf{e}_r = [G\int_{V_t}\frac{(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{r}}')}{\|\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{r}}'\|^3}\frac{\Phi_m}{c^2}r'dr'd\theta ds]\mathbf{e}_r$
$= T(t)\frac{GK}{c^2}\int_{V_t}\frac{(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{r}}')\mathbf{e}_r}{\|\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{r}}'\|^3}\frac{1}{(r'^2+s^2)^{1/4}}r'dr'd\theta ds$
$= T(t)\frac{GK}{c^2}\int_{-r_0}^{r_0}ds\int_0^{r_0(t,s)}dr'\frac{r'}{(r'^2+s^2)^{1/4}}\int_0^{2\pi}\frac{(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{r}}')\mathbf{e}_r d\theta}{\|\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{r}}'\|^3}$
$= T(t)\frac{GK}{c^2}\int_{-r_0}^{r_0}ds f(t,x,r,s)$
where
$f(t,x,r,s) = \int_0^{r_0(t,s)}dr'\frac{r'}{(r'^2+s^2)^{1/4}}\int_0^{2\pi}\frac{(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{r}}')\mathbf{e}_r d\theta}{\|\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{r}}'\|^3}$
and from $(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{r}}')\mathbf{e}_r = r'\cos\theta - r$ (here $\mathbf{e}_r$ is the unit radial vector in the point defined by $\overrightarrow{\mathbf{r}}$) and $\|\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{r}}'\|^3 = (x^2 + r^2 + s^2 + r'^2 - 2xs - 2rr'\cos\theta)^{3/2}$, we obtain (23).
□
For $t = 0$, with $T(0) = 1$ and $u_T(0) = 0$ in (22), and with $r_0(0,x) = \sqrt{r_0^2 - x^2}$, we obtain the solution of the hydrostatic equilibrium when the internal radial force (*pressure-gradient force* per unit of area) inside particle's body $\overrightarrow{F} = -\frac{\partial P(0,x,r)}{\partial r}\mathbf{e}_r = \mathbf{e}_r\frac{K}{c^2\sqrt{\rho}}\int_{-r_0}^{r_0}\frac{KG}{c^2}f(0,x,r,s)ds = \frac{8\pi GK^2}{5c^4}\mathbf{e}_r > 0$ is balanced by the self-gravitational force oriented into particle's barycenter [8]. However, for $t > 0$, in the case of this 3-D symmetry breaking expansion (when $k < 0$), we have also the first term on the right-hand side of (22), created by the presence of the large massive barrier which caused this extremely strong perturbation and generated a radial density speed $u_r(t,x,r) = u_T(t)u_R(x,r) = -\frac{2}{3}k\frac{\rho^2}{r}u_T(t) > 0$ with the acceleration $\frac{d}{dt}u_r(t,x,r) = \frac{du_T(t)}{dt}u_R(x,r) + u_T^2u_R(x,r)\frac{\partial u_R}{\partial r} = -\frac{2}{3}k\frac{\rho^2}{r}(\frac{du_T(t)}{dt} - \frac{2k}{3}(1 - \frac{x^2}{r^2})u_T^2(t))$. Thus, now the internal force $\overrightarrow{F}(t,x,r)$ is not more constant in each point of particle's body but changes (by considering the strong impact into the barrier, so that the first term in the right-side of (22) is more than one

order dominant w.r.t. to the self-gravitational force represented by the second term) proportionally to $\frac{\rho^{3,2}}{r} \approx \sqrt{r}$ for $r >> r_0$ (in each fixed time-instance $t > 0$ if $\frac{du_T(t)}{dt} \neq 0$). That is, we have the major changes of the internal force at the parts of particle's body which are more far from particle's barycenter, as expected.
The Lemma 1, based on the momentum conservation law, is intended to be used for the derivation of the disk expansion $r_0(t,x)$, and it can be seen if we replace $T(t)$ and $u_T(t)$ from the equation (17) into the equation (22). However, this complex method will be presented in more details in the next section, where will be also analyzed the methods, results and provided the conclusions.
Let us finish this section by considering now, from the physical point of view, a complete cycle of particle's auto-equilibrium dynamics when a particle passes from an inertial propagation with a stationary distribution $\Phi_m = \frac{K}{\sqrt{\rho}}$ into an explosive energy-density cylindrical disk expansion caused by 3-D symmetry breaking with enormously large massive barrier *without* any slit (the case of the barrier with the slits will be considered in the next section). Then, after some interval of time, when all kinetic energy (which the particle had before the physical impact onto this barrier) is spent for the radial expansion and hence when this disk expansion is interrupted, now, after $\triangle t$, becomes the inverse process of the disk compression (now with $k > 0$). Now, during this compression, the internal potential energy transforms into the kinetic energy but with opposite direction of the particle's propagation, because it cant enter into the massive barrier (case of the perfect elastic reflection from the barrier), and hence this particle again returns into the inertial stationary propagation:

- **Disk expansion process**: It happens when, at $t = 0$, a particle, from its initial inertial stationary propagation with energy-density distribution in the sphere with initial radius r_0, $\Phi_m(0,x,r) = \frac{K}{\sqrt{\rho}} = \frac{K}{(r^2+x^2)^{1/4}}$, with $r^2 + x^2 \leq r_0^2$, starts to be decelerated (by the 3-D symmetry breaking caused by ideally infinite barrier orthogonal to the particle's trajectory) with conversion of the particle's kinetic energy into its internal potential energy. This internal potential energy increments the internal pressure force $-\frac{\partial P}{\partial r}$ which now becomes enormously greater than particles self-gravitational force. So, now begins the process of particle's radial expansion (parallel to the surface of this barrier) with the velocity $\overrightarrow{\mathbf{w}} = \overrightarrow{\mathbf{u}} = u_r(t,x,r)\mathbf{e}_r = u_T(t)u_R(x,r)\mathbf{e}_r = -\frac{2}{3}k\frac{\rho^2}{r}u_T(t)\mathbf{e}_r$ (with $k < 0$).
  During this particle's deceleration, i.e., the time interval $0 < t < \triangle t$, this stationary energy density changes, as a result of the radial expansion ve-

locity $u_r(t,x,r)$, and creates a disk-shape between the plane $x = -r_0$ and the plane $x = r_0$. In this case, at the end of this expansion (when all particle's kinetic energy is converted into its internal potential energy V so that we have the complete rest of this macro-disk of the particle at the barrier surface, with $r_0(\triangle t, 0)$ enormously bigger than r_0), we obtain that the density of the particle's body diminished and becomes equal to

$$\Phi_m(\triangle t, x, r) = T(\triangle t)\frac{K}{(r^2+x^2)^{1/4}} = \frac{1}{e^{|k|\int_0^{\triangle t} u_T(s)ds}}\frac{K}{\sqrt{\rho}} < \frac{K}{\sqrt{\rho}} \qquad (25)$$

for $-r_0 \le x \le r_0$ and $r \le r_0(\triangle t, x)$.
During this period of the expansion (when $k < 0$), the first term on the right-hand side of the internal force $-\frac{\partial P}{\partial r}$ (in the equation (22)) is positive as well. However, at the end of this time interval, i.e., at $t = \triangle t$, when the internal force of expansion disappears, also the expansion acceleration disappears, that is $\frac{du_T(t)}{dt}|_{t=\triangle t} = 0$. So, in this instance of time, the internal force becomes $\overrightarrow{F}(\triangle t, x, r) = -\frac{\partial P(\triangle t, x, r)}{\partial r}$ much more smaller than at an instance of time before $\triangle t$. Consequently, the self-gravitational force becomes greater than this internal pressure and hence now begins the process of particles compression (with the speed $u_r(t,x,r) = -\frac{2}{3}k\frac{\rho^2}{r}u_T(t)$ for $t > \triangle t$ becomes negative (now with $k > 0$)).

- **Self-compression process**: Let us consider now what happens for $t > \triangle t$, when the self-gravitational forces inside particle's disk become greater than the internal pressure force and hence begins the process of the disk compression. During compression, the previously accumulated internal potential energy now converts into the kinetic energy (which was zero at $t = \triangle t$) and so begins the process of the perfect reflection of the particle from the barrier. The energy-density distribution of the particle at $t = \triangle t$ is given by (25). Consequently, we have that during the expansion (for $0 < t < \triangle t$), $T(t) = e^{k\int_0^t u_T(s)ds} = e^{-|k|\int_0^t u_T(s)ds} < 1$, while now for $t > \triangle t$, $T(t) = e^{k\int_0^t u_T(s)ds} = e^{|k|\int_0^t u_T(s)ds} > 1$, and hence now $\Phi_m(t,x,r) = T(t)\frac{K}{\sqrt{\rho}}$ increases with time. That is, the particle's density now increases during this compression phase. So, for $t > \triangle t$, $T(t) > 1$ and hence $r_0(t,x)$ now decreases, confirming that we obtained a compression of the particle's body with the velocity $\frac{dr_0(t,x)}{dt} = u_r(t,x,r_0(t)) = u_T(t)u_R(r_0(t),x) = u_T(t)(-\frac{2}{3}k\frac{r_0^2(t,x)+x^2}{r_0(t,x)}) < 0$ because now, for $t > \triangle t$, we have $k > 0$ and hence the radial velocity is negative (oriented toward the particle's barycenter), as expected.

So, now we have an inverse process, that is, a compression of the particle's body up to the radius r_0 when the distribution of the energy-density again becomes time-invariant $\Phi_m(t,r) = \frac{K}{\sqrt{\rho}}$. Thus, we obtain again a stationary particle's state in which the internal forces become constant in each point inside particle's body and hence the internal energy-flow velocity $\overrightarrow{\mathbf{w}} = \overrightarrow{\mathbf{u}} = u(t,r)\mathbf{e}_r$ becomes zero again. Thus, we return again to particle's hydrostatic equilibrium[4], as it was in the initial moment $t = 0$ before the particle's expansion caused by an acceleration, with *the same* (but in opposite direction) speed of this particle. In fact, the whole internal potential energy V, which at $t = \triangle t$ was equal to particle's initial kinetic energy before the impact with the barrier, now is again completely returned to particle's kinetic energy without any kind of energy dissipation, as explained from the perfect elastic properties of particle's perfectly fluid body.

In the time instance $t = \triangle t$, we have the rest of the radial expansion, because the whole kinetic energy of the particle now is converted into the internal potential energy used to extend the particle's body against particle's internal self-gravitational forces, Thus, now we can use the conservation of internal energy law (6) which, from the fact that $\Phi_m(t,x,r) = T(t)\frac{K}{\sqrt{r^2+x^2}}$ and $\nabla \cdot \overrightarrow{\mathbf{u}} = u_T(t)\frac{\partial u_R(x,r)}{\partial r} = u_T(t)\frac{\partial}{\partial r}(-\frac{2k}{3}\frac{r^2+x^2}{r}) = -\frac{2k}{3}u_T(t)(1-\frac{x^2}{r^2})$, reduces to

$$\frac{de(t,x,r)}{dt} = \frac{2kc^2}{3K}\sqrt{r^2+x^2}(1-\frac{x^2}{r^2})\frac{u_T(t)}{T(t)}P(t,x,r) \tag{26}$$

Thus, the particle's kinetic energy E_K (before the impact with the large massive barrier (by considering that $\frac{de(t,x,r)}{dt}$ is negative during particle's expansion) is given by,

$$E_K = -\int_0^{\triangle t} dt \int_{V_t} \frac{de(t,x,r)}{dt} dV \tag{27}$$

where V_t is the volume of particle's body at time t, for $0 \le t \le \triangle t$ and, by considering our cylindrical coordinate system, the infinitesimal volume is $dV = rdrdxd\theta$. Hence, from (26) and (27), we obtain the following equation for the kinetic energy of the particle, in relationship with the internal particle's pressure

[4]It will be necessary also to investigate the damping ratio: the eventual oscillatory behavior when the particle after acceleration returns into its inertial propagation with internal hydrostatic equilibrium. Does we have an exponential decrease as a function of time (analog to Landau damping, for example) of such spherical particle's density oscillatory waves has to be investigated.

$P(t, x, r)$ inside the particle's body during the whole process of particle's radial expansion (when $k < 0$):

$$E_K = -\frac{4\pi kc^2}{3K}\int_0^{\triangle t}\frac{u_T(t)}{T(t)}dt\int_{-r_0}^{r_0}dx\int_0^{r_0(t,x)} r\sqrt{r^2+x^2}(1-\frac{x^2}{r^2})P(t,x,r)dr \quad (28)$$

In next section, we will consider the method for computation of the radial extension of particle's disk $r_0(t, x)$ in order to analyze for which conditions this radial expansion would generate a very large macro-dimensional disk (in an extremely short interval of time δt), able to reach both slits in the large massive barrier, and to propagate trough them to reach the other side of barrier, by generating the typical effects of the double-slit experiments [6] also in the massive particles cases.

4 Conclusion: Application to double-slit experiments: a test of the IQM theory

The radial expansion of the particle's disk-body for a given x, such that $-r_0 \leq x \leq r_0$, and time-instance $t \geq 0$, is given by $r_0(t, x)$, with $r_0(0, x) = \sqrt{r_0^2 - x^2}$, so that $r_0(0, 0) = r_0$ is the radius of the particle's sphere in its hydrostatic equilibrium. In this section, we will analyze the methods for the computation of this dynamic disk extension, which is not so important for the case of the perfect reflection of the particle from the large massive barrier, as explained at the end of the previous section, but is fundamental in consideration of the double-slit experiments. We consider the massive barrier with two slits divided by the macroscopic distance d, enormously greater than the radius r_0 of the massive elementary particle in its hydrostatic equilibrium (as it was examined in [6], Section 4.6.1, but without providing the method of computation of the radial extension of the massive particle when it comes near to the barrier, approximatively in the center between these two slits).
This method for computation of $r_0(t, x)$ is necessary in order to demonstrate mathematically that it is possible to obtain the real phenomena of the radial explosion of the massive particle such that, after extremely short interval of time δt (from particles hydrostatic equilibrium, in the time-instance $t = 0$), we can have that $r_0(\delta t, x) > \frac{d}{2} >> r_0$ and hence the possibility that particle's body passes contemporarily trough both slits.
In effect, from the equation (17) in Proposition 2, we have that both time-dependent functions $T(t)$ and $u_T(t)$ (the dynamic components of the particle's

density and radial velocity time-changing) does not depend on x (which appears in the right-hand sides of (17)), so that we can rewrite the equation (17) for the simplest case when $x = 0$:

$$T(t) = (\frac{r_0}{r_0(t,0)})^{3/2} \leq 1, \quad and \quad u_T(t) = -\frac{3}{2k}\frac{1}{r_0(t,0)}\frac{dr_0(t,0)}{dt} \geq 0 \qquad (29)$$

Consequently, by dividing the first equations in (17) and (29) with $T(t)$, we obtain:

$$r_0^2(t,x) = r_0^2(t,0) - x^2 \quad \leq r_0^2(t,0) \qquad (30)$$

Thus, we have the maximum of extension of the disk in the plane $x = 0$, which passes through particle's barycenter (the middle of the disk) and, moreover, the particle's disk has the form of the part of the sphere with radius $r_0(t,0)$ between the two plains $x = -r_0$ and $x = r_0$. Thus, for $r_0(t,0) >> r_0$ the border of this disk will be practically flat surface during fast radial propagation.
If we repeat now the same division of the second equations in (17) and (29) with $u_T(t)$, we obtain:

$$\frac{dr_0(t,x)}{dt} = \frac{r_0(t,0)}{\sqrt{r_0^2(t,0) - x^2}}\frac{dr_0(t,0)}{dt} \quad \geq \frac{dr_0(t,0)}{dt} \qquad (31)$$

So, also the radial velocity of the disk expansion, for any $-r_0 \leq x \leq r_0$, can be obtained from its radial velocity at $x = 0$ and its radial expansion $r_0(t,0)$, such that the minimal velocity of particle's radial expansion is just in the particle's barycenter plane $x = 0$ and maximal for $|x| = r_0$, i.e., in the front and back surfaces of particle's disk.
Thus, it is enough to find a more simple method for the computation, only of the maximal disk extension in the center of the disk (for $x = 0$), that is, the computation of $r_0(t,0)$. Consequently, we can simplify the equations in Lemma 1, obtained from the momentum conservation law, as follows:

Corollary 1 *The equation (22) for the boundary surface of the particle's disk in the plane $x = 0$, when $r = r_0(t,0)$, is reduced into*

$$\int_{-r_0}^{r_0} f(t,0,r_0(t,0),s)ds = \frac{-c^2}{KGr_0^{3/2}}(\frac{c^2}{K}\frac{r_0^{7/2}(t,0)}{r_0^{3/2}}\frac{\partial P(t,0,r)}{\partial r}|_{r=r_0(t,0)} + r_0^{3/2}(t,0)\frac{dr_0^2(t,0)}{dt^2}) \qquad (32)$$

for the function $f(t,x,r,s)$ defined by (23). This equation then reduces to the *following 3rd order differential equation for* $r_0(t,0)$,

$$(\int_{-a}^{a} dq \int_0^{2\pi} \frac{d\theta}{(1-\sqrt{1-q^2}\cos\theta)^{1/2}})\frac{dr_0(t,0)}{dt}$$
$$= \frac{c^2\sqrt{8}}{KGr_0^{3/2}}\sqrt{r_0(t,0)}\frac{d}{dt}(\frac{c^2}{K}\frac{r_0^{7/2}(t,0)}{r_0^{3/2}}\frac{\partial P(t,0,r)}{\partial r}|_{r=r_0(t,0)} + r_0^{3/2}(t,0)\frac{dr_0^2(t,0)}{dt^2}) \quad (33)$$

where $a = r_0/r_0(t,0) \le 1$.

Proof:
1. The proof of (32): for $x=0$ and $r=r_0(t,0)$, so that $\rho=\sqrt{r^2+x^2}=r=r_0(t,0)$, the (22) reduces to
$-\frac{\partial P(t,0,r)}{\partial r}|_{r=r_0(t,0)} =$
$T(t)\frac{K}{c^2\sqrt{r_0(t,0)}}[-\frac{2k}{3}r_0(t,0)(\frac{du_T}{dt}-\frac{2k}{3}u_T^2(t)) + \int_{-r_0}^{r_0} T(t)\frac{KG}{c^2}f(t,0,r_0(t,0),s)ds]$.
Thus, by considering that $T(t)=(\frac{r_0}{r_0(t,0)})^{3/2}$ and $u_T(t)=-\frac{3}{2k}\frac{1}{r_0(t,0)}\frac{dr_0(t,0)}{dt}$ (from (17) when $x=0$), so that $\frac{2k}{3}(\frac{du_T}{dt}-\frac{2k}{3}u_T^2(t)) = -\frac{1}{r_0(t,0)}\frac{d^2r_0(t,0)}{dt^2}$,
we obtain
$\int_{-r_0}^{r_0} f(t,0,r_0(t,0),s)ds$
$=\frac{c^2r_0(t,0)}{KGT(t)}[-\frac{c^2}{KT(t)\sqrt{r_0(t,0)}}\frac{\partial P(t,0,r)}{\partial r}|_{r=r_0(t,0)} + \frac{2k}{3}(\frac{du_T}{dt}-\frac{2k}{3}u_T^2(t))]$
$=\frac{c^2r_0^{5/2}(t,0)}{KGr_0^{3/2}}[-\frac{c^2r_0(t,0)}{Kr_0^{3/2}}\frac{\partial P(t,0,r)}{\partial r}|_{r=r_0(t,0)} + \frac{2k}{3}(\frac{du_T}{dt}-\frac{2k}{3}u_T^2(t))]$
$=\frac{c^2r_0^{5/2}(t,0)}{KGr_0^{3/2}}[-\frac{c^2r_0(t,0)}{Kr_0^{3/2}}\frac{\partial P(t,0,r)}{\partial r}|_{r=r_0(t,0)} - \frac{1}{r_0(t,0)}\frac{d^2r_0(t,0)}{dt^2}]$
$=-\frac{c^2}{KGr_0^{3/2}}(\frac{c^2}{K}r_0^{7/2}(t,0)\frac{\partial P(t,0,r)}{\partial r}|_{r=r_0(t,0)} + r_0^{3/2}(t,0)\frac{dr_0^2(t,0)}{dt^2})$

2. The proof of (33): from (23) by applying Leibnitz integral rule, we have
$\frac{d}{dt}\int_{-r_0}^{r_0} dsf(t,0,r_0(t,0),s)$
$=\frac{d}{dt}(\int_{-r_0}^{r_0} ds\int_0^{r_0(t,s)} dr'\frac{r'}{(r'^2+s^2)^{1/4}}\int_0^{2\pi}\frac{(r'\cos\theta-r_0(t,0))d\theta}{(r_0^2(t,0)+s^2+r'^2-2r_0(t,0)r'\cos\theta)^{3/2}})$
$=\int_{-r_0}^{r_0} ds\frac{dr_0(t,s)}{dt}(\frac{r_0(t,s)}{(r_0^2(t,s)+s^2)^{1/4}}\int_0^{2\pi}\frac{(r_0(t,s)\cos\theta-r_0(t,0))d\theta}{(r_0^2(t,0)+s^2+r_0^2(t,s)-2r_0(t,0)r_0(t,s)\cos\theta)^{3/2}})$
$=\int_{-r_0}^{r_0} ds(-\frac{2ku_T(t)}{3})(r_0^2(t,s)+s^2)^{3/4}\cdot$
$\int_0^{2\pi}\frac{(r_0(t,s)\cos\theta-r_0(t,0))d\theta}{(r_0^2(t,0)+s^2+r_0^2(t,s)-2r_0(t,0)r_0(t,s)\cos\theta)^{3/2}}$ from (17), so from $r_0(t,s)=\sqrt{r_0^2(t,0)-s^2}$
$=-\frac{2ku_T(t)}{3}\int_{-r_0}^{r_0} dsr_0^{3/2}(t,0)\int_0^{2\pi}(\frac{1}{2r_0(t,0)})^{3/2}\frac{(\sqrt{r_0^2(t,0)-s^2}\cos\theta-r_0(t,0))d\theta}{(r_0(t,0)-\sqrt{r_0^2(t,0)-s^2}\cos\theta)^{3/2}}$
$=(\frac{1}{2})^{3/2}\frac{2ku_T(t)}{3}\int_{-r_0}^{r_0} ds\int_0^{2\pi}\frac{d\theta}{(r_0(t,0)-\sqrt{r_0^2(t,0)-s^2}\cos\theta)^{1/2}}$
$=(\frac{1}{2})^{3/2}\frac{2ku_T(t)}{3}\sqrt{r_0(t,0)}\int_{-r_0}^{r_0}\frac{ds}{r_0(t,0)}\int_0^{2\pi}\frac{d\theta}{(1-\sqrt{1-(s/r_0(t,0))^2}\cos\theta)^{1/2}}$

$= \frac{1}{\sqrt{8}} \frac{2ku_T(t)\sqrt{r_0(t,0)}}{3} \int_{-a}^{a} dq \int_0^{2\pi} \frac{d\theta}{(1-\sqrt{1-q^2}\cos\theta)^{1/2}}$ for $q = \frac{s}{r_0(t,0)} \leq 1$
$= -\frac{1}{\sqrt{8}} (\int_{-a}^{a} dq \int_0^{2\pi} \frac{d\theta}{(1-\sqrt{1-q^2}\cos\theta)^{1/2}}) \frac{1}{\sqrt{r_0(t,0)}} \frac{dr_0(t,0)}{dt}$ from (17) when $x = 0$.
So, from (32) and this equation above, we obtain (33).
□
It is easy to verify that (33) reduces to the following 3rd order differential equation for the maximal disk's extension $r_0(t,0)$:

$$(\int_{-a}^{a} dq \int_0^{2\pi} \frac{d\theta}{(1-\sqrt{1-q^2}\cos\theta)^{1/2}}) \frac{dr_0(t,0)}{dt}$$
$$= \frac{c^4\sqrt{8}}{K^2 G r_0^3} (\frac{d}{dt} (\frac{\partial P(t,0,r)}{\partial r}|_{r=r_0(t,0)}) r_0^4(t,0) + \frac{7}{2} \frac{\partial P(t,0,r)}{\partial r}|_{r=r_0(t,0)} r_0^3(t,0) \frac{dr_0(t,0)}{dt})$$
$$+ \frac{c^2\sqrt{8}}{K G r_0^{3/2}} (\frac{3}{2} r_0(t,0) \frac{dr_0^2(t,0)}{dt^2} + r_0^2(t,0) \frac{dr_0^3(t,0)}{dt^3}) \quad (34)$$

Consider now the particle's kinetic energy in (28), from the fact that $\frac{u_T(t)}{T(t)} = -\frac{3}{2k} \frac{\sqrt{r_0(t,0)}}{r_0^{3/2}} \frac{dr_0(t,0)}{dt}$ and $r_0(t,x) = \sqrt{r_0^2(t,0) - x^2}$ for $-r_0 \leq x \leq r_0$, which becomes an expression depending only on the dynamical radial extension at plain $x = 0$, $r_0(t,0)$, and on the pressure $P(t,x,r)$,

$$E_K = \frac{\pi c^2}{K r_0^{3/2}} \int_0^{\triangle t} \frac{dr_0(t,0)}{dt} \sqrt{r_0(t,0)} dt$$
$$\int_{-r_0}^{r_0} dx \int_0^{\sqrt{r_0^2(t,0)-x^2}} r\sqrt{r^2+x^2}(1-\frac{x^2}{r^2}) P(t,x,r) dr \quad (35)$$

So, after more than 7 years of the development of the *completion of QM*, by the missed part of theory dedicated to an *individual* particle (and not to an ensemble with its statistical theory and probability densities based on the Schrödinger equation and its relativistic Dirac's extension), in which it is provided also the calculation of the *mass-spectrum* (Section 2.3 in [6]) of the elementary particles (which can not be obtained from the statistical-probabilistic theory of Standard Model), we are now able to *verify experimentally* this new part of the QM for the individual massive elementary particles, described as complex 3-D wave-packets Ψ such that $\Phi_m = \Psi\overline{\Psi}$ is the *rest-mass energy density.*
In fact, we can not measure this rest-mass energy density (it is a hidden, non-observable variable) and we can not observe the internal speed of this particle's density during the particle's accelerations. This new theory in a number of sig-

nificant examples [6] is in accordance with the results of the current ensemble-based statistical QM, and it holds also for the famous double-slit example[5]. However, the practical examples of 'how we can test this new theory by laboratory experiments' are not provided in [6], because the mathematical elaboration of the physical process of a radial explosion into a macro-disk of a massive elementary particle in front of the barrier was not developed in physically enough details, but only up to a mathematical level which permits the definition of the new non-Poincare group of transformations ([6], Section 1.3), which is a more specific algebraic QM theoretical issue. In this paper, we present in details this missed physical part, which is not strictly a (standard) QM issue, but more appropriate to the *classical* (and deterministic) mechanics, that is, to the general theory of fluids [1][6] with the fundamental difference that the particle's body is not structured as a molecular-composed fluids but as *a perfect fluid*.

It is well known that Landau went to Copenhagen on 8 April 1930 to work at the Niels Bohr's Institute for Theoretical Physics. He stayed there until 3 May of the same year. After the visit, Landau always considered himself a pupil of Niels Bohr and Landau's approach to physics was greatly influenced by Bohr (he worked also with Wolfgang Pauli, another Copenhagen's school follower). It is something mysterious for me that he was one of the greatest scientists just in the classical mechanics (theory of fluids in this case relevant to my work), as Schrödinger, and hence he received the 1962 Nobel Prize in Physics for his development of a mathematical theory of superfluidity, but it seems that he did not tried to overcome the Bohr's Copenhagen-interpretation of QM by completing the QM and unify it with classical mechanics as wanted Albert Einstein (and I proposed in [6]: so-called *Landau quasiparticles* can be physically explained as unstable fermions whose part of their kinetic energy is converted into an internal potential energy (negative if particle is spherically extended from its hydrostatic equilibrium, positive if it is compressed [8]), so that their mass can differ substantially from that of a normal fermion). Probably because he did not work never with Einstein, Max Planck or Schrödinger, the main opposition to the Copenhagen's interpretation? Hope to find in next to this open question.

However, in the mathematical framework developed in this work, we are now

[5]Richard Feynman [2] was fond of saying that all of quantum mechanics can be gleaned from carefully thinking through the implications of this single experiment

[6]In 1924, Landau moved to the main centre of Soviet physics at the time: the Physics Department of Leningrad State University (Saint Petersburg University, great Russian school, heredity of great Leonhard Euler, which theory of fluids I used here; Euler arrived in Saint Petersburg on 17 May 1727, invited by Peter the Great to improve education in Russia and to close the scientific gap with Western Europe in that historical period) where he dedicated himself to the study of theoretical physics, graduating in 1927.

able to define the following experimental tests for this new IQM theory of an individual particle:

1. In order to obtain the double-slit interference for the electrons, the kinetic energy of each individual test electron must be inside an interval of values $[E_{min}, E_{max}]$. The values of the kinetic energy lower than E_{min} are too much low in order to obtain enough internal potential energy able to expand the particle's disk over both slits.
   That is, with such a low kinetic energies we would have that $r_0(\triangle t, 0) < \frac{d}{2}$, where d is the distance between two slits in the massive barrier. The kinetic energy greater than E_{max} will not produce the effect of enough radial extension, because the particle would have a very high speed toward the barrier and hence the time for the radial extension (when particle is infinitesimally near to the barrier) would be too much short for the significant radial extension. In a such situation, an electron would have the direct collision with some elementary particle inside the barrier and would be blocked inside this barrier.

2. For a greater distance between two slits, the value E_{min} must be higher and this fact can be experimentally checked by the measurements of E_{min} in the case of a number of different distances d between two slits.

Another test of this new theory for radial expansion of the particle, during the 3-D symmetry breaking caused by the (very) large massive barrier, can be also provided theoretically by the numerical computations of the radial expansion $r_0(t, 0)$, based on the differential equations (33) and (35).
For example, for a given distance d between two slits, we can measure the minimal value of the kinetic energy E_{min} for which the extended disk of the electron does not reach the slits and hance would not be enable to pass through them and to achieve the inference in the back side of the barrier. Thus, we can set $E_K = E_{min}$ in (35) and $r_0(\triangle t, 0) = \frac{d}{2}$ for the solution of (33). So, we are able to apply, for example, the variational method of computation, by assuming that the internal pressure $P(\triangle t, 0, r_0(\triangle t, 0))$ is equal to that of the hydrostatic equilibrium for the radius $r_0(\triangle t, 0)$, and then to use the variational numerical methods for (33).

Conclusion and future work:
In 1915, Einstein published four groundbreaking papers that introduced his theory of general relativity. But at the time, the German-born theoretical physicist

was hardly known to the public, and members of the science community were fighting his new theory head-on, according to reports in the New York Times. Around this time, Sir Isaac Newton's model of classical mechanics (formulated in his 1687 book "Philosophi Naturalis Principia Mathematica") ruled, and Einstein's work was met with utmost skepticism. Based on calculations Einstein made, about his new theory of general relativity, light from another star should be bent by the Sun's gravity. In 1919, that prediction was confirmed by Sir Arthur Eddington during the solar eclipse of 29 May 1919. Those observations were published in the international media, making Einstein world famous.
Unfortunately for me, I have no a cathedra in physics and hence no access to the quantum laboratory equipped for the double-slit experiments with electrons. Moreover, I have only the laptops as working instruments without the necessary numerical tools for the approximative solutions of the two differential equations above.
So, I hope that the more experienced and well equipped quantum physicists can consider seriously these tests and provide the definite answers
Such a research work would be well suited for some PhD student thesis as well and I will be glad, if invited, to offer some of my advise for it. So, if there is some candidate PhD student or research group, please be free to contact me for such a possibility at my e-mail address.

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