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WEAK SHADOWING PROPERTY IN Ω-STABLE DIFFEOMORPHISMS

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Abstract.

The weak shadowing property was introduced by R.M. Corless and S.Yu. Pilyugin and studied by these authors, K. Sakai, O.B. Plamenevskaya and others. It was shown by Plamenevskaya that for omega-stable diffeomorphisms this property may be bount to the numerical properties of the eigenvalues of the hyperbolic saddle points of the diffeomorphisms.

In this paper, we prove that if the phase diagram of an omega-stable diffeomorphism of a manifold does not contain chains of length more than three, then it has the weak shadowing property.

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1 Introduction

The weak shadowing property of dynamical systems was introduced in [1], where it was shown that this property is C^0-generic.

The study of the weak shadowing property for Ω-stable diffeomorphisms is essentially complicated: it was shown by Plamenevskaya [2] (see below) that this property may be bount to the numerical properties of the eigenvalues of hyperbolic saddle points of the diffeomorphisms.

In this paper we prove Theorem 2.1 stating that Ω-stable diffeomorphisms (on manifolds of arbitrary dimension) having only "short" connections in phase diagrams have the weak shadowing property.

2 Definitions and main results

Let M be a closed smooth manifold with Riemannian metric dist. Denote by $U(a, A)$ the a-neighborhood of a set $A \subset M$.

Denote by $\mathrm{Diff}^1(M)$ the space of diffeomorphisms of M with the C^1 topology. For a diffeomorphism f, we denote by $O(x, f)$ the trajectory of x.

A sequence $\xi = \{x_k : k \in \mathbb{Z}\} \subset M$ is called a *d-pseudotrajectory of f* if

$$\mathrm{dist}(f(x_k), x_{k+1}) < d, \quad k \in \mathbb{Z}.$$

We say that a point $x \in M$ *ϵ-shadows* the pseudotrajectory ξ if

$$\mathrm{dist}(f^k(x), x_k) < \epsilon, \quad k \in \mathbb{Z}.$$

We say that a point $x \in M$ *weakly ϵ-shadows* ξ if

$$\xi \subset U(\epsilon, O(x, f)).$$

Now we give definitions of the main properties which we study.

We say that a diffeomorphism f has the (*usual*) *shadowing property* if, given $\epsilon > 0$, there exists $d > 0$ such that any d-pseudotrajectory is ϵ-shadowed by some point of M.

We say that f has the *weak shadowing property* if, given $\epsilon > 0$, there exists $d > 0$ such that any d-pseudotrajectory is weakly ϵ-shadowed by some point of M.

Remark 2.1. Let us note that the property defined above was called the first weak shadowing property in [3], where the second weak shadowing property, "symmetric" to the first one, was introduced: we say that f has the second weak shadowing property if, given $\epsilon > 0$, there exists $d > 0$ such that for any d-pseudotrajectory ξ of f, there is a point x such that

$$O(x, f) \subset U(\epsilon, \xi).$$

It was shown in [3] that any dynamical system with compact phase space has the second weak shadowing property, hence the study of this property in the context of our paper is senseless. For this reason, we use below the term "weak shadowing property" introduced in [1].

Of course, if a diffeomorphism has the shadowing property, it has the weak shadowing property as well. An example of irrational rotation on the circle shows that the inverse statement does not hold.

The following example constructed by Plamenevskaya [2] gives us useful information concerning weak shadowing in Ω-stable systems.

Example. Represent $\mathbb{T}^2$ as the square $[-2, 2] \times [-2, 2]$ with identified opposite sides. Let $g : \mathbb{T}^2 \to \mathbb{T}^2$ be a diffeomorphism with the following properties:

(**1**) the nonwandering set $\Omega(g)$ of g is the union of 4 hyperbolic fixed points; that is, $\Omega(g) = \{p_1,\ p_2,\ p_3,\ p_4\}$, where p_1 is a source, p_4 is a sink, and $p_2,\ p_3$ are saddles;

(**2**) with respect to coordinates $(v, w) \in [-2, 2] \times [-2, 2]$, the following conditions hold:

(2.1) $p_1 = (1, 2),\ p_2 = (1, 0),\ p_3 = (-1, 0),\ p_4 = (-1, 2)$,

(2.2) $W^u(p_2) \cup \{p_3\} = W^s(p_3) \cup \{p_2\} = [-2, 2] \times \{0\}$,

$$W^s(p_2) = \{1\} \times (-2, 2),\ W^u(p_3) = \{-1\} \times (-2, 2),$$

where $W^s(p_i)$ and $W^u(p_i)$ are the stable and unstable manifolds, respectively, defined as usual;

(2.3) there exist neighborhoods $U_2,\ U_3$ of $p_2,\ p_3$ such that

$$g(x) = p_i + D_{p_i}g(x - p_i) \ \text{ if } \ x \in U_i,$$

(2.4) there exists a neighborhood U of the point $z = (0, 0)$ such that $g(U) \subset U_3,\ g^{-1}(U) \subset U_2$ and g^{-1} is affine on $g(U)$, (2.5) the eigenvalues of $D_{p_3}g$ are $-\mu,\ \nu$ with $\mu > 1,\ 0 < \nu < 1$, and the eigenvalues of $D_{p_2}g$ are $-\lambda,\ \kappa$ with $0 < \lambda < 1,\ \kappa > 1$.

It was proved in [2] that g has the weak shadowing property if and only if the number $\log\lambda/\log\mu$ is irrational. Note that g satisfies Axiom A and the no-cycle condition (i.e., it is Ω-stable) but does not have the shadowing property.

Let f be an Axiom A diffeomorphism of M. By the Smale Spectral Decomposition Theorem, the nonwandering set $\Omega(f)$ can be represented as a finite union of basic sets Ω_i. Denote by $W^s(\Omega_i)$ and $W^u(\Omega_i)$ the stable and unstable "manifolds" of Ω_i. For two different basic sets Ω_i and Ω_j, we write $\Omega_i \to \Omega_j$ if

$$W^u(\Omega_i) \cap W^s(\Omega_j) \neq \emptyset.$$

Let us say that the phase diagram of the diffeomorphism f contains a *chain of length* m if there exist m different basic sets $\Omega_{i_1}, \dots, \Omega_{i_m}$ such that

$$\Omega_{i_1} \to \dots \to \Omega_{i_m}.$$

Theorem 2.1. *Assume that a diffeomorphism f satisfies Axiom A and the no-cycle condition. If its phase diagram does not contain chains of length $m > 3$, then f has the weak shadowing property.*

Note that the restriction on the lengths of chains in Theorem 2.1 is sharp: the Ω-stable diffeomorphism in the Plamenevskaya example has a chain $p_1 \to p_2 \to p_3 \to p_4$ of length 4 in its phase diagram (and may fail to have the weak shadowing property).

3 Proof of Theorem 2.1

Let us first introduce some notation.

Denote by $O_+(x, f)$ and $O_-(x, f)$ the positive and negative semitrajectories of x, respectively. Let $\xi = \{x_k : k \in \mathbb{Z}\}$ be a pseudotrajectory and let l, m be indices with $l \le m$. We denote

$$\xi^{l,m} = \{x_k : l \le k \le m\}, \quad \xi^l_+ = \{x_k : l \le k\}, \quad \xi^l_- = \{x_k : k \le l\},$$
$$\xi_+ = \xi^0_+, \text{ and } \xi_- = \xi^0_-.$$

The following three propositions are well known (Proposition 3.1 is the classical Birkhoff theorem, for proofs of statements similar to Propositions 3.2 and 3.3, see [4], for example).

Proposition 3.1. *Let f be a homeomorphism of a compact topological space X and U be a neighborhood of its nonwandering set. Then there exists a positive number N such that*

$$\text{card}\{k : f^k(x) \notin U\} \leq N$$

for any $x \in X$, where card A *is the cardinality of a set A.*

In Propositions 3.2, 3.3, 3.2p and 3.3p, we assume that f is an Ω-stable diffeomorphism of a closed smooth manifold (below we apply these propositions both to f and f^{-1}).

Proposition 3.2. *If Ω_i is a basic set, then for any neighborhood U of Ω_i we can find its neighborhood V with the following property: if for some $x \in V$ and $m > 0$, $f^m(x) \notin U$, then $f^{m+k}(x) \notin V$ for $k \geq 0$.*

Proposition 3.2. *There exist neighborhoods U_i of the basic sets Ω_i such that if $f^m(U_i) \cap U_j \neq \emptyset$ for some $m > 0$, then there exist basic sets $\Omega_{l_1}, \dots, \Omega_{l_k}$ such that*

$$\Omega_i \to \Omega_{l_1} \to \cdots \to \Omega_{l_k} \to \Omega_j.$$

Obviously, these propositions have the following analogs for pseudotrajectories.

Proposition 3.1p. *Let f be a homeomorphism of a compact metric space X and U be a neighborhood of its nonwandering set. Then there exist positive numbers d, N such that if $\xi = \{x_k\} \subset X$ is a d-pseudotrajectory and $\xi^{l,m} \cap U = \emptyset$ for some l, m with $l \leq m$, then $m - l \leq N$.*

Proposition 3.2p. *If Ω_i is a basic set, then for any neighborhood U of Ω_i we can find its neighborhood V and a number $d > 0$ with the following property: if $\xi = \{x_k\}$ is a d-pseudotrajectory of f, $x_0 \in V$, and $x_m \notin U$ for some $m > 0$, then $\xi_+^m \cap V = \emptyset$.*

Proposition 3.3p. *There exist neighborhoods U_i of the basic sets Ω_i and a number $d > 0$ with the following property: if $\xi = \{x_k\}$ is a d-pseudotrajectory of f such that $x_0 \in U_i$ and $x_m \in U_j$ for some $m > 0$, then there exist basic sets $\Omega_{l_1}, \dots, \Omega_{l_k}$ such that*

$$\Omega_i \to \Omega_{l_1} \to \cdots \to \Omega_{l_k} \to \Omega_j.$$

In what follows, we assume that f is an Ω-stable diffeomorphism. We need the following auxiliary statement. Let us say that f has the *usual shadowing property on a set* A if, given $\epsilon > 0$, there exists $d > 0$ such that if $\xi = \{x_k\}$ is a d-pseudotrajectory of f with $\xi^{l,m} \subset A$, then there exists x such that $\text{dist}(x_k, f^k(x)) < \epsilon$ for $l \le k \le m$. Since any basic set Ω_i is hyperbolic, we may assume that f has the usual shadowing property on all neighborhoods of Ω_i considered below.

Lemma. *Let Ω_i be a basic set and let U_i be a neighborhood of Ω_i such that $\overline{U}_i \cap \Omega_j = \emptyset$ for $i \ne j$. For any positive α, there exists $d > 0$ with the following property: if $\xi = \{x_k\}$ is a d-pseudotrajectory of f with $\xi_+ \subset U_i$, then there exists a point z and an open set D containing z such that*

(1) $\text{dist}(x_0, z) < \alpha$;

(2) $\xi_+ \subset U(\alpha, O_+(z', f))$ *for any* $z' \in D$.

Proof. Fix arbitrary $\alpha > 0$. Reducing α, if necessary, we may assume that

$$\overline{U(\alpha, U_i)} \cap \Omega_j = \emptyset$$

for $j \ne i$. Applying the usual shadowing property on U_i, let us find $d > 0$ such that if $\xi = \{x_k\}$ is a d-pseudotrajectory of f with $\xi_+ \subset U_i$, then there exists y such that $\text{dist}(x_k, f^k(y)) < \alpha/4$ for $k \ge 0$. By the choice of α, $O_+(y, f) \subset U(\alpha, U_i)$, hence $y \in W^s(\Omega_i)$. Thus, there exists $p \in \Omega_i$ such that $y \in W^s(p)$. In any neighborhood of p, there is a point q such that its trajectory is dense in Ω_i. Stable manifolds of points of a hyperbolic set depend continuously on the point, hence any neighborhood of y contains a point z such that $O_+(z, f)$ is dense in Ω_i.

There exists a number $K > 0$ such that $f^k(y) \in U(\alpha/4, \Omega_i)$ for $k \ge K$. Find a point z such that

(1) $\text{dist}(f^k(y), f^k(z)) < \alpha/2$ for $0 \le k \le K$;

(2) $O_+(z, f)$ is dense in Ω_i.

There exists a number $L > 0$ such that for any point $p \in \Omega_i$ there is a point $r \in \{f^k(z) : 0 \le k \le L\}$ with $\text{dist}(p, r) < \alpha/4$. By the continuity of f, there is an open set D containing z such that $\Omega_i \subset U(\alpha/2, O_+(z', f))$ for any $z' \in D$.

To complete the proof, it remains to take D so small that $\text{dist}(f^k(y), f^k(z')) < \alpha/2$ for $0 \le k \le K$ and $z' \in D$.

Remark 3.1. Let Ω_i be an attractor. Fix $\epsilon > 0$ and find a neighborhood U_i of Ω_i such that

$$U_i \subset U(\epsilon/2, \Omega_i) \tag{1}$$

and $f(\overline{U}_i) \subset U_i$. There exist numbers $d, a > 0$ (depending only on U_i) such that if $\xi = \{x_k\}$ is a d-pseudotrajectory of f with $x_0 \in U_i$, then $\xi_+ \subset U_i$, there is a point $y \in W^s(\Omega_i)$ such that $\operatorname{dist}(f^k(y), x_k) < \epsilon/4$, and $W = U(a, x_1) \subset U_i$. Since points z for which $O_+(z, f)$ is dense in Ω_i are dense in W, the same reasoning as in the proof of the lemma above shows that the set

$$W' = \{x \in W : \xi_+^1 \subset U(\epsilon, O_+(x, f))\}$$

is open and dense in W.

Of course, a similar statement holds for a repeller Ω_i.

In the proof of Theorem 2.1, we have to consider d-pseudotrajectories with decreasing values of d. We use the same notation of points of these pseudotrajectories, of their neighborhoods, etc; this will lead to no confusion.

Let m be the maximal length of chains in the phase diagram of the considered Ω-stable diffeomorphism f. If there are no chains of length 2, then the statement of our theorem is trivial – in this case, f is an Anosov diffeomorphism.

Let us consider the case where $m = 2$. In this case, any basic set is either a repeller or an attractor. Consider a repeller Ω_1 and an attractor Ω_2. Fix an arbitrary $\epsilon > 0$. Standard reasons show that there exist neighborhoods U_i of the sets Ω_i, $i = 1, 2$, such that inclusions (1) hold, $f^{-1}(\overline{U}_1) \subset U_1$, and $f(\overline{U}_2) \subset U_2$.

The set $U_2' = f(\overline{U}_2) \setminus f^2(U_2)$ is a compact subset of U_2 disjoint from Ω_2. Hence, there exists a number $a_2 \in (0, \epsilon)$ and a neighborhood V_2 of Ω_2 such that

$$U(a_2, x) \subset U_2 \setminus V_2$$

for any $x \in U_2'$.

Similarly, there exists a number $a_1 \in (0, \epsilon)$ and a neighborhood V_1 of Ω_1 such that

$$U(a_1, x) \subset U_1 \setminus V_1$$

for any $x \in U_1' = f^{-1}(\overline{U}_1) \setminus f^{-2}(U_1)$.

We may assume that these numbers and neighborhoods have also the following properties. There exists a number $d_1 > 0$ such that if $\xi = \{x_k\}$ is a d_1-pseudotrajectory of f and $x_m \in U_2$, then $\xi_+^m \subset U_2$ and, in addition, if $x_{m-1} \notin U_2$, then

$$U(a_2, x_m) \subset U_2 \setminus V_2$$

(and similar statements hold for U_1 etc).

It follows from Propositions 3.1p-3.3p that there exist numbers $d_2 \in (0, d_1)$ and N such that if $\xi = \{x_k\}$ is a d_2-pseudotrajectory of f, then only one of the following possibilities holds:

(I) there exists a basic set Ω_i such that $\xi \subset U_i$;

(II) there exists a repeller Ω_1 and an attractor Ω_2 such that, for the neighborhoods described above, there exist integers l, m with $0 \le m - l \le N$ such that
$$U(a_1, x_l) \subset U_1 \setminus V_1 \text{ and } U(a_2, x_m) \subset U_2 \setminus V_2.$$

Case (I) is trivial since a basic set contains a dense trajectory (and, by condition (3.1), ξ belongs to the ϵ-neighborhood of such a trajectory).

To consider case (II), find positive numbers $a_3 < a_1$ and $d_3 < d_2$ such that for any points x, y with $\operatorname{dist}(x, y) < a_3$ and for any d_3-pseudotrajectory $\{y_k\}$ with $y_0 = y$, the inequalities
$$\operatorname{dist}(f^k(x), y_k) < a_2$$
hold for $0 \le k \le N$.

Let $\xi = \{x_k\}$ be a d_3-pseudotrajectory such that
$$U(a_1, x_l) \subset U_1 \setminus V_1 \text{ and } U(a_2, x_m) \subset U_2 \setminus V_2$$
for some l, m with $0 \le m - l \le N$. Denote $W_1 = U(a_3, x_l)$ and $W_2 = U(a_2, x_m)$.

The remark after the lemma implies that a_3, a_2, d_3 can be chosen in such a way that the sets
$$W_1' = \{x \in W_1 : \xi_-^l \subset U(\epsilon, O_-(x, f))\}$$
and
$$W_2' = \{x \in W_2 : \xi_+^m \subset U(\epsilon, O_+(x, f))\}$$
are open and dense subsets of W_1 and W_2, respectively.

By our choice of d_3, $f^{m-l}(W_1) \subset W_2$. Since $f^{m-l}(W_1')$ is an open and dense subset of $f^{m-l}(W_1)$, there is a point $x' \in f^{m-l}(W_1') \cap W_2'$.

Take $x = f^{l-m}(x')$. It is easy to see that
$$\xi_-^l \subset N(\epsilon/2, O_-(x, f)), \quad \xi_+^m \subset N(\epsilon/2, O_+(x, f)),$$

and $\text{dist}(f^{k-l}(x), x_k) < \epsilon$ for $l \le k \le m$, hence $\xi \subset U(\epsilon, O(x, f))$. This completes the consideration of the case $m = 2$.

Finally, we consider the case $m = 3$. Fix $\epsilon > 0$. It follows from Propositions 3.1p – 3.3p that there exist numbers $d_0, N > 0$ and neighborhoods U_i of the basic sets Ω_i such that inclusions (1) hold and, for any d_0-pseudotrajectory $\xi = \{x_k\}$ of f, only one of the following possibilities is realized:

(P1) there exists an index i such that $\xi \subset U_i$;

(P2) there exist a repeller Ω_i, an attractor Ω_j, and indices l, m with $l < m$ such that $m - l \le N$, $\xi_-^l \subset U_i$, and $\xi_+^m \subset U_j$;

(P3.1) there exist a repeller Ω_i, a saddle basic set (i.e., a basic set that is not an attractor or repeller) Ω_j, and indices l, m with $l < m$ such that $m - l \le N$, $\xi_-^l \subset U_i$, and $\xi_+^m \subset U_j$;

(P3.2) there exist a saddle basic set Ω_i, an attractor Ω_j, and indices l, m with $l < m$ such that $m - l \le N$, $\xi_-^l \subset U_i$, and $\xi_+^m \subset U_j$;

(P4) there exist a repeller Ω_i, a saddle basic set Ω_j, an attractor Ω_s, and indices l, m, n, t with $l < m < n < t$ such that $m - l \le N$, $t - n \le N$, $\xi_-^l \subset U_i$, $\xi^{m,n} \subset U_j$, and $\xi_+^t \subset U_s$.

For possibilities (P1) and (P2), the proof is just the same as in the case $m = 2$.

Let us consider possibility (P3.1) (the same reasoning is applicable for (P3.2)). Similarly to the proof for the case $m = 2$, we can find $a_i, d_1 > 0$ such that, for any d_1-pseudotrajectory ξ with $x_l \in U_i$, $W_i = U(a_i, x_{l-1}) \subset U_i$. After that, we find $a_j \in (0, \epsilon)$ and $d_2 < d_1$ such that for any d_2-pseudotrajectory ξ with $x_l \in U_i$, $x_m \in U_j$, and $0 \le m - l \le N$, the inclusion

$$W_j = U(a_j, x_m) \subset f^{m-l+1}(W_i)$$

and the inequalities $\text{dist}(f^{k-m}(y), x_k) < \epsilon$ hold for any $y \in W_j$ and $l \le k \le m$.

Applying the lemma (with $\alpha = a_j$), we can find $d_3 < d_2$ with the following property: for any d_3-pseudotrajectory ξ there exists an open subset D of W_j such that $\xi_+^m \subset U(\epsilon, O_+(z, f))$ for any $z \in D$.

Applying the remark after the lemma, we may assume that, for any d_3-pseudotrajectory ξ, the set

$$W_i' = \{x \in W_i : \xi_-^{l-1} \subset U(\epsilon, O_-(x, f))\}$$

is open and dense in W_i. Its image, $f^{m-l+1}(W_i')$, contains W_j (and hence, there is a point x belonging to the intersection of this image with the open subset D of W_j).

It follows from our constructions that $\xi \subset U(\epsilon, O(x,f))$. This completes the consideration of possibility (P3.1).

Finally, we have to consider possibility (P4). Fix a repeller Ω_i, a saddle basic set Ω_j, and an attractor Ω_s for which there exists a d_0-pseudotrajectory ξ and indices l,m,n,t with $l < m < n < t$ such that $m-l \le N$, $t-n \le N$, $\xi_-^l \subset U_i$, $\xi^{m,n} \subset U_j$, and $\xi_+^t \subset U_s$.

We may assume that the neighborhoods U_i, U_j, U_s satisfy inclusions (1). In addition, we assume that for d_2-pseudotrajectories with $d_2 < d_1$ and for numbers $a_i, a_s \in (0,\epsilon)$, all of the statements similar to statements in the proof for the case $m = 2$ (II) are valid (with natural replacement of U_1, U_2, etc by U_i, U_s, etc).

To be exact, we assume that if ξ is a d_2-pseudotrajectory with $\xi_-^l \subset U_i$, $\xi^{m,n} \subset U_j$, and $\xi_+^t \subset U_s$, then the sets $W_i = U(a_i, x_l)$ and $W_s = U(a_s, x_t)$ are subsets of U_i and U_s, respectively, and that the sets

$$W_i' = \{x \in W_i : \xi_-^l \subset U(\epsilon, O_-(x,f))\}$$

and

$$W_s' = \{x \in W_s : \xi_+^t \subset U(\epsilon, O_+(x,f))\}$$

are their open and dense subsets.

Now let us find numbers $d_3 < d_2$ and $a_j > 0$ such that, for any point x_r of a d_3-pseudotrajectory ξ and for any point y such that $\operatorname{dist}(y, x_r) < a_j$, the inequalities

$$\operatorname{dist}(f^k(y), x_{r+k}) < \min(a_i, a_s)$$

hold if $|k| \le N$.

In addition, since f has the usual shadowing property on U_j and $\xi^{m,n} \subset U_j$, we may assume that there exists a point y such that $\operatorname{dist}(f^k(y), x_{m+k}) < a_j$ for $0 \le k \le n-m$ (note that the value $n-m$ may be arbitrarily large, in contrast to the values $m-l$ and $t-n$ not exceeding N).

Denote $W_{j,1} = U(a_j, x_m)$ and $W_{j,2} = U(a_j, x_n)$. By the choice of a_j, $V_i = f^{l-m}(W_{j,1}) \subset W_i$ and $V_s = f^{t-n}(W_{j,2}) \subset W_s$. Hence, the intersection $V_i' = V_i \cap W_i'$ is open and dense in V_i, and the intersection $V_s' = V_s \cap W_s'$ is open and dense in V_s. It follows that the image $V' = f^{m-l}(V_i')$ is open and dense in $W_{j,1}$, and the image $V'' = f^{n-t}(V_s')$ is open and dense in $W_{j,2}$.

It remains to note that the point y has a small neighborhood $D \subset W_{j,1}$ such that $f^{n-m}(D) \subset W_{j,2}$ and $\mathrm{dist}(f^k(x), x_{m+k}) < \epsilon$ for $x \in D$ and $0 \leq k \leq n-m$. It follows from our considerations that there exists a point $x \in D \cap V'$ such that $f^{n-m}(x) \subset V''$. By construction, $\xi \subset U(\epsilon, O(f,x))$.

The theorem is proved.

Remark 3.2. Analyzing the proof of Theorem 2.1, it is easy to see that a similar statement holds for an Ω-stable diffeomorphism f under the following condition: if

$$\Omega_i \to \Omega_{l_1} \to \cdots \to \Omega_{l_k} \to \Omega_j$$

is a chain in the phase diagram of f such that Ω_i is a repeller and Ω_j is an attractor, then stable and unstable manifolds of points of the basic sets $\Omega_{l_1}, \dots, \Omega_{l_k}$ are transverse.

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