DIFFERENTIAL EQUATIONS
AND
CONTROL PROCESSES
N. 4, 2020
Electronic Journal,
reg. N ΦC77-39410 at 15.04.2010
ISSN 1817-2172

http://diffjournal.spbu.ru/
e-mail: jodiff@mail.ru

Control problems in nonlinear systems
Optimal control
Dynamic economic models

Dynamics of monetary and fiscal policy interaction in New Keynesian model in continuous time

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Abstract. The paper studies monetary and fiscal policy rules consistent with determinate equilibrium dynamics. We consider a three-dimensional New Keynesian model in continuous time with Rotemberg price-setting mechanism and non-Ricardian consumers, and study dynamics of the model under monetary and fiscal policies interactions. The key analytical finding is that the uniqueness of the equilibrium solution can be never achieved while both policies are in their active regimes. Another combinations of policies' regimes can lead to a local equilibrium determinacy through control of appropriate values of Taylor coefficients (f^M and f^F). We demonstrate that in contrast to economy with Ricardian consumers, making government debt a net asset leads to creation of an additional channel for fiscal and monetary policy interaction and changing the conditions for local equilibrium determinacy. In addition, we show that in case of explosive equilibrium dynamics, limit cycle or more complicated attracting sets could appear, including chaotic attractors of various natures.

Key words: New Keynesian model, local equilibrium determinacy, global indeterminacy, chaos, nonlinear dynamics, hidden oscillation, fiscal and monetary policies' interaction, Taylor rule.

1 Introduction

Mainstream macroeconomic analysis of both monetary and fiscal policies relies heavily on the New Keynesian model. New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models are typically set as systems of discrete time expectational difference equations [16, 45–47]. In contrast, typical continuous time models in economics are deterministic dynamical systems, which complicates consideration of the stochastic nature of economic development. Recently, there appeared several papers with DSGE models set in continuous time [12, 19]. One reason for this is that model in continuous time might be more suitable for analytical investigation than the corresponding discrete time variant [20].

Traditionally, economists used discrete-time models for description of events taking place at business cycle frequencies (see, e.g. [8, 35, 36]), while continuous time modelling was restricted to the growth theory [15, 39, 44]. Still, for certain purposes discrete-time and continuous-time models could be used interchangeably and lead to similar model predictions. For example, in [21] it is pointed out that considering very short time periods in discrete-time models (for instance, one day instead of a quarter) could lead to the same qualitative results as in the corresponding continuous time model.

Recent monetary policy literature presents an example of a traditional (discrete time) field that started to investigate implications of continuous time modeling: a whole subfield of the Heterogeneous Agents New Keynesian (HANK) modeling depends on derivation and aggregation of the individual agents' decision rules, and is facilitated by the usage of continuous time modeling [12, 19, 20, 23]. HANK appeared as conjunction of two directions: heterogeneous-agent (HA) and New Keynesian (NK). In this framework researchers typically use quantitative models (for instance, DSGE) involving heavy machinery for their resolution. However, it is important to build simple and tractable models to gain analytical insights into their underlying mechanisms and make their policy conclusions sharper and easier to undestand [9, 10, 37]. Investigation of effects of monetary and fiscal policies in liquidity trap (see, e.g., [52]) benefited from ability to derive closed form results in a deterministic continuous time model.

We study a continuous-time model similar to [30]. This is a small New Keynesian model in which consumers face a constant probability of death that makes government debt a net asset of the agents, introducing non-Ricardian features. In contrast to Leith and von Thadden [30], we assume that the firms set prices according to Rotemberg price-setting mechanism [42] rather than that of Calvo [13]. Making this assumption allows to derive the exact nonlinear NK Phillips curve and avoid linearization of the model.

The main contribution of the paper is to analyze local equilibrium determinacy[1] of considered model under different monetary and fiscal policies regimes. We also consider policy regimes combinations leading to explosive equilibrium; as we derive the model in the exact nonlinear form, we are able to characterize the nature of global indeterminacy that might appear in such a case.

In modern macroeconomics local equilibrium determinacy is often analyzed as being influenced by the monetary and/or fiscal policy in the framework of a standard NK model. It is important to empathize that NK models are predominantly used in studies that analyze the effects of monetary policy on the local equilibrium determinacy. Several papers (see, e.g. [25, 43]) demonstrate that local equilibrium determinacy is always ensured under an active monetary policy rule (when monetary authority is aimed at stabilizing inflation). Nevertheless, in [22], for instance, it is shown that there are some restrictions for monetary authorities on policy rules which fail to guarantee the determinacy of local equilibrium. Authors consider simple interest rate rule as well as other alternative forms of them in a standard NK model in continuous time. As a result, they derive a set of parameters where the equilibrium is not determinate under active monetary policy. Moreover, it is postulated in [22] that monetary policy should not be very active in order to guarantee the determinacy of local equilibrium. The paper confirms results of [14, 17] and demonstrates that “activeness” of monetary policy rule does not necessary mean avoidance of indeterminacy. Moreover, all these papers provide results that multiple local equilibria can be achieved in case of combination of endogenous capital accumulation and sticky prices.

However, all these studies do not take into consideration fiscal aspects. One of the earliest works where local equilibrium determinacy is studied under effects

[1] If an equilibrium is determinate, there is unique solution for the model's variables. Thus, locally determinate equilibrium is (locally) unique.

of both monetary and fiscal policy rules is [32]. Leeper constructs a stochastic maximizing model in discrete time and shows that local equilibrium determinacy is achieved, when active monetary policy interacts with passive fiscal policy and vice versa. Later other studies confirmed Leeper's results. For instance, analysing optimal monetary and fiscal rules in NK model in discrete time, Schmitt-Grohe and Uribe [48] find out that active monetary and passive fiscal policies ensure the uniqueness of the rational expectations equilibrium. In contrast to [32], they consider a model with price stickiness, including money and distortionary taxation.

Kumhof and co-authors [29] study the influence of policies' rules on a unique equilibrium from the side of fiscal dominance: situation when fiscal authorities do not control tax revenues and spending. They consider model in discrete time with transaction cost technologies and price stickiness and conclude that when fiscal policy is active, monetary policy should also react to public debt in order to expand the set of unique equilibrium and improve welfare. The main message of this study is that fiscal policy does influence local equilibrium determinacy and it is a matter of great importance to maintain fiscal discipline before introduction of inflation targeting.

Discussed above papers study the uniqueness of equilibrium and policies' interaction using discrete time NK models. The same results are also confirmed in studies where NK approach in continuous time models is considered [3, 5, 6, 51]. For example, Tsuzuki [51] develops NK model in continuous time and studies influence of monetary and fiscal policies' lag on local equilibrium determinacy. He constructs a delay-differential equation system that models the economy with money-in-the-production function and finds out effects from policies' interaction on equilibrium. The main finding is that policy lags do matter in achieving local equilibrium determinacy. Adding policy lag can lead to resolving of indeterminacy under the combination of active monetary and passive fiscal policies. Moreover, it is pointed out that monetary and fiscal authorities can act as substitutions. In other words, for achieving determinacy in case indeterminacy occurs it is sufficient to shorten or lengthen a policy lag in only one policy rule.

Leith and von Thadden [30] consider NK model in continuous time with capital accumulation and non-Ricardian consumers. They study simple monetary and fiscal policy rules that lead to the determinacy of local equilibrium. They find that the level of government debt influences the analysis of equilibrium dynamics. Moreover, for ensuring a determinate equilibrium it is necessary to determine the level of the government debt. Depending on "high" or "low" debt the form of monetary and fiscal interaction that ensures the equilibrium determinacy is chosen. It is demonstrated in [30] that threshold values of steady-state debt associate with bifurcation parameters and it leads to qualitative changes in requirements for local equilibrium determinacy.

Locally determinate equilibrium ensures that the solution for the agents' choice variables is unique, and that their expectations are uniquely determined, as well. Therefore, it is highly desired that economic policy delivered a determinate equilibrium. Alternatively, investigation of policies that fail to achieve determinate equilibrium is important for providing guidance to the policymakers. In this regard, the analysis could be broadened to further investigation of a set of parameters that can lead to periodic, quasiperiodic and chaotic dynamics (including hidden oscillations) [1, 2, 18, 26, 27, 34]. It was suggested that business cycles may be largely driven by endogenous cyclical forces. In particular, Benhabib et. al. [7] found that backward-looking interest rate feedback rules do not guarantee uniqueness of equilibrium and present examples in which for plausible parameterizations of the economy attracting equilibrium cycles exist. Barnett et. al. [3] provide an evidence that in a New Keynesian (NK) model, an active interest rate feedback monetary policy, when combined with a Ricardian passive fiscal policy, may induce occurrence of a Shilnikov chaotic attractor in the region of the parameter space where uniqueness of the equilibrium prevails locally. Beaudry et. al. [4] examine the notion of stochastic limit cycles, wherein the system is buffeted by exogenous shocks, but where the deterministic part of the system admits a limit cycle. Usage of continuous time in models with endogenous cycles might be advantageous, as such models could be analyzed through developed tools of mathematical control theory and stochastic differential equations.

Our study expands on the literature by considering a model with Rotemberg pricing (see, e.g., [31]), which allows to derive exactly a more compact system of equations better suited to investigation of dynamic interactions between the fiscal and monetary policies. In our analysis we consider passive and active regimes for both policies as per classification introduced by Leeper [32]. The main findings of the paper are as follows. First, local equilibrium determinacy cannot obtain when both policies are active. Second, local equilibrium determinacy in any other policies' combinations depends on ranges of f^F and f^M and can be achieved only through appropriate choice of values of these parameters. Third, in contrast to Ricardian equivalence, considering government debt as a net asset leads to a creation of an additional channel for fiscal and monetary policy interaction and changing conditions for local equilibrium determinacy.

The rest of the paper is organized as follows. Section 2 describes the theoretical model with Rotemberg price-setting mechanism and finitely lived consumers. Section 3 provides analysis of the system's local equilibrium determinacy under different policies' regimes, the case for global indeterminacy, calibration of parameters and graphical analysis. Then, Section 4 concludes.

2 The model

The basic structure of the economy is considered as in [30]. The model describes the economy where there are four main economic agents, namely representative consumer, representative producer, the government and Central bank (CB). The representative consumer maximizes its intertemporal utility function constrained by the budget through choosing the level of consumption and labor. Operating in monopolistic market, the representative producer maximizes its profit. The producer rents labor to manufacture goods and changes prices following Rotemberg price-setting mechanism [42]. The government issues bonds, collects lump-sum taxes and consumes. CB fixes one-period nominal interest rate. The government and CB are responsible for fiscal and monetary policies respectively. In what follows, we set the problem for every agent and describe agents' activities.

2.1 The representative consumer problem

We consider the optimization problem:

$$\max_{C_t^j, N_t^j} \int_0^\infty \left(\ln C_t^j - \frac{(N_t^j)^{1+\varphi}}{1+\varphi} \right) e^{-(\xi+\rho)t} dt. \tag{1}$$

Each period of time t the consumer j faces the following utility function $U_t^j = \ln C_t^j - \frac{(N_t^j)^{1+\varphi}}{1+\varphi}$, where the value of his utility depends on goods he consumes C_t^j, the level of labor he supplies N_t^j, and φ is the inverse of the labor supply elasticity. The utility function is discounted by the probability of death ξ and time preferences ρ. Simultaneously, the representative consumer faces the following budget constraint:

$$\dot{b}_t^j = (i_t - \pi_t + \xi)\, b_t^j + \frac{W_t}{P_t} N_t^j - \tau_t^j - C_t^j + \Omega_t^j. \tag{2}$$

Budget constraint demonstrates that at each time t the consumer j obtains real income from his labor activity $\frac{W_t}{P_t} N_t^j$ (where W_t is nominal wage, P_t is a price level), returns on bonds $(i_t - \pi_t)\, b_t^j$ and transfers from firms Ω_t^j. At the same time the consumer uses all his income for consumption C_t^j, paying taxes to the government τ_t^j and increasing or decreasing his bonds $\dot{b}_t^j$. Moreover, it is assumed that all bonds of deceased consumers are transferred to the currently alive consumers, including newborn. In such a way the term ξb_t^j refers to the profit that can be gained by alive consumers from deceased consumers.

Hamiltonian function for considered consumer optimization problem takes the form

$$H_t(C_t^j, N_t^j, \dot{b}_t^j, \lambda_t) = \\ C_t^j - \frac{(N_t^j)^{1+\varphi}}{1+\varphi} + \lambda_t \left[(i_t - \pi_t + \xi)\, b_t^j + \frac{W_t}{P_t} N_t^j - \tau_t^j - C_t^j + \Omega_t^j \right], \tag{3}$$

where C_t^j and N_t^j are control variables, λ_t is a co-state variable, b_t^j is a state variable.

The first-order conditions (FOC) derived through Hamiltonian can be written as follows:

$$\frac{\partial H_t}{\partial C_t^j} = \frac{1}{C_t^j} - \lambda_t = 0, \tag{4}$$

$$\frac{\partial H_t}{\partial N_t^j} = \left(N_t^j\right)^{\varphi} - \lambda_t \frac{W_t}{P_t} = 0, \tag{5}$$

$$\frac{\partial H_t}{\partial b_t^j} = \lambda_t\,(i_t - \pi_t - \xi) = \lambda_t\,(\xi + \rho) - \dot{\lambda}_t. \tag{6}$$

The combination of conditions (4) and (5) reveals the relation between consumption, labor, level of wages and prices:

$$C_t^j (N_t^j)^{\varphi} = \frac{W_t}{P_t}. \tag{7}$$

The dynamics of consumption is attained then by differentiating condition (4) and combining it with condition (6):

$$\frac{\dot{C}_t^j}{C_t^j} = i_t + \rho - \pi_t, \tag{8}$$

where inflation is defined as $\pi_t = \frac{\dot{p}_t}{p_t}$.

Taking the next step, we apply a transversality condition regarding all non-human (b_t^j) and human (h_t^j) wealth are equal to total consumption of the consumer

j (C_s^j) discounted at the real interest rate $i_t - \pi_t$ plus the probability of death ξ:

$$\int_t^{\infty} C_s^j e^{-\int_t^s (i_\mu - \pi_\mu + \xi) d\mu} ds = b_t^j + h_t^j, \tag{9}$$

where $h_t^j = \int_t^{\infty} \left(\frac{W_s N_s^j}{P_s} - \frac{T_s^j}{P_s} + \Omega_t^j \right) e^{-\int_t^s (i_\mu - \pi_\mu + \xi) d\mu} ds$. Integrating by parts the transversality condition (9) and applying relation of consumption (8) we obtain $C_t^j = (\xi + \rho) \left(b_t^j + h_t^j \right)$. Aggregating to all consumers the latter relation, we obtain the following expression:

$$C_t = (\xi + \rho) (b_t + h_t). \tag{10}$$

Moreover, aggregation of human and non-human wealth dynamics gives us:

$$\dot{h}_t = (i_t - \pi_t + \xi) h_t - \frac{W_t N_t}{P_t} + \frac{T_t}{P_t} - \Omega_t, \tag{11}$$

$$\dot{b}_t = (i_t - \pi_t) b_t + \frac{W_t N_t}{P_t} - C_t - \frac{T_t}{P_t} + \Omega_t. \tag{12}$$

As a result, the dynamics of aggregated consumption is expressed by following equation:

$$\dot{C}_t = (i_t - \pi_t - \rho) C_t - \xi (\xi + \rho) b_t. \tag{13}$$

2.2 The representative producer problem

In the economy we consider the monopolistically competitive producer facing Rotemberg [42] costs of price changes in every period. According to this, he has to pay quadratic adjustment costs of price changes. These costs are set as follows:

$$\Theta \left(\frac{\dot{p}_t^i}{p_t^i}, t \right) = \frac{\theta}{2} \left(\frac{\dot{p}_t^i}{p_t^i} \right)^2 P_t Y_t, \tag{14}$$

where θ measures the degree of price stickiness.

In order to eliminate resource costs of inflation we make an assumption that adjustment costs of inflation are paid by producer as transfers to consumers (the term Ω_t in the consumer's optimization problem). Taking into consideration quadratic adjustment costs, we formulate the producer optimization problem:

$$V_0(p_0) = \max_{p_t, t \geq 0} \int_0^{\infty} e^{-\int_0^t i_s(s) ds} \left(\Pi_t (p_t) - \frac{\theta}{2} \left(\frac{\dot{p}_t^i}{p_t^i} \right)^2 P_t Y_t \right) dt, \tag{15}$$

where $\Pi_t(p_t) = p_t^i \left(\frac{p_t^i}{P_t}\right)^{-\varepsilon} Y_t - \frac{W_t}{A_t}\left(\frac{p_t^i}{P_t}\right)^{-\varepsilon} Y_t$ reflects profits per each period of time.

Demand function every producer i faces at time t takes the form:

$$y_t^i = \left(\frac{p_t^i}{P_t}\right)^{-\varepsilon} Y_t.$$

Moreover, in the considered economy (economy without capital) the production function is constructed by labor N_t^i multiplied by the technology A_t: $y_t^i = A_t N_t^i$. For derivation of dynamical equation for inflation (Phillips curve) we formulate Hamiltonian function, where state variable is p_t, control variable is $\dot{p}_t$ and co-state variable is η_t :

$$H(p_t, \dot{p}_t, \eta_t) = p_t^i \left(\frac{p_t^i}{P_t}\right)^{-\varepsilon} Y_t - \frac{W_t}{A_t}\left(\frac{p_t^i}{P_t}\right)^{-\varepsilon} - \frac{\theta}{2}\left(\frac{\dot{p}_t^i}{p_t^i}\right)^2 P_t Y_t + \eta_t \dot{p}_t. \tag{16}$$

Conditions for optimum derived through optimization are as follows:

$$\frac{\partial H}{\partial \dot{p}_t} = -\theta \frac{\dot{p}_t^i}{p_t^i}\frac{P_t}{p_t^i} Y_t + \eta_t = 0, \tag{17}$$

$$\frac{\partial H}{\partial p_t^i} = (1-\varepsilon)\left(\frac{p_t^i}{P_t}\right)^{-\varepsilon} Y_t + \varepsilon \frac{W_t}{p_t^i}\frac{1}{A_t}\left(\frac{p_t^i}{P_t}\right)^{-\varepsilon} Y_t + \theta \left(\frac{\dot{p}_t^i}{p_t^i}\right)^2 \frac{P_t}{p_t^i} Y_t = -\dot{\eta}_t + i_t \eta_t. \tag{18}$$

Assuming symmetric equilibrium $\left(p_t^i = P_t\right)$ we rearrange terms and rewrite conditions, defining $\pi_t = \frac{\dot{p}_t^i}{p_t^i}$:

$$\theta \pi_t Y_t = \eta_t, \tag{19}$$

$$\dot{\eta}_t = i\eta_t - \left[(1-\varepsilon) Y_t + \varepsilon \frac{W_t}{P_t}\frac{1}{A_t} Y_t + \theta \pi_t^2 Y_t\right]. \tag{20}$$

Differentiating η_t from (19) with respect to time and substituting it into equation (20), we get the relation of optimal price-setting:

$$\left(i_t - \pi_t - \frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\varepsilon - 1}{\theta}\left(\frac{\varepsilon}{\varepsilon - 1}\frac{W_t}{A_t P_t} - 1\right) + \dot{\pi}_t. \tag{21}$$

Taking the next step, we consider that in case of flexible prices (supposing there are no adjustment costs $\theta = 0$) the prices are derived from the following condition: $\Pi_t'(p_t) = 0$. From this we obtain the "natural" level of prices:

$$P_t^n = \frac{\varepsilon}{\varepsilon - 1}\frac{W_t^n}{A_t}, \tag{22}$$

where index n means the natural rate. As in a usual model of monopolist price setting, the monopolist's price is higher than the marginal cost (higher than $\frac{W_t}{A_t}$ in our case). The gross markup is given by $\frac{\varepsilon}{\varepsilon-1}$.

In order to bring the level of output to the socially optimal level, the government could pay a subsidy to either price (thus paying it to the consumers) or to the wage (therefore, making the firms the subsidy's recipients). In the latter case, denoting the gross subsidy R and taking into account (22), the firm will pay the wage equal to W_t/R and the optimality condition are the following

$$P_t^n = \frac{\varepsilon}{\varepsilon - 1}\frac{W_t^n/R}{A_t}. \tag{23}$$

If the gross subsidy is such that $R = \frac{\varepsilon}{\varepsilon-1}$, then the usual competitive condition of the price equals to the marginal cost obtains.

Applying consumer's FOC (7), market clearing condition $Y_t = C_t$ and the production function expressed as $A_t = \frac{Y_t}{N_t}$, we obtain the following substitution:

$$\frac{W_t}{A_t P_t} = Y_t^{1+\varphi} A_t^{-1-\varphi} = N_t^{1+\varphi} = C_t^{1+\varphi}.$$

Finally, making the same assumption as in the flexible prices case that the government is responsible for gross subsidy to wages $R = \frac{\varepsilon}{\varepsilon-1}$, we repeat the derivation of the firms' optimality condition by replacing W_t with W_t/R to obtain the Phillips curve derived from the producer's problem as

$$\dot{\pi}_t = \left(r_t - \frac{\dot{Y}_t}{Y_t}\right)\pi_t - \frac{\varepsilon - 1}{\theta}\left(\frac{\varepsilon}{\varepsilon - 1}\frac{W_t/R}{A_t P_t} - 1\right) = \left(r_t - \frac{\dot{Y}_t}{Y_t}\right)\pi_t - \frac{\varepsilon - 1}{\theta}\left(C_t^{1+\varphi} - 1\right). \tag{24}$$

Moreover, from market clearing condition and dynamical equation for consumption

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{C}_t}{C_t} = (r_t - \rho) - \xi(\xi + \rho)\frac{b_t}{C_t}.$$

As a result, the dynamics of inflation is expressed as follows:

$$\dot{\pi}_t = \left(\rho + \xi(\xi + \rho)\frac{b_t}{C_t}\right)\pi_t - \frac{\varepsilon - 1}{\theta}\left(C_t^{1+\varphi} - 1\right). \tag{25}$$

2.3 The government and fiscal policy

Dynamics of the public sector liabilities are defined by the following equation:

$$\dot{b}_t = r_t b_t + g_t - \tau_t. \tag{26}$$

Government expenditures in terms of aggregate final output are fixed ($g_t = g > 0$). Moreover, it is assumed that the government expenditures g include part of the wages in the economy paid out as a subsidy. With productivity equal to one at all times we obtain $N = Y = C$. Subsidy wage rate equals marginal product of labor which equals one. Therefore, the government pays subsidy $\left(1 - \frac{1}{R}\right) = \frac{1}{\varepsilon}$ per unit of output produced, for a total subsidy of $\frac{C}{\varepsilon}$. In order to make the model economically meaningful, it necessary to set the level of government spending g such that $g > \frac{C}{\varepsilon}$.

Comparing to the debt dynamics (12) we obtain the GDP by income identity. It demonstrates that in the economy private plus government consumption is equal to total household resources (wages plus transfers from producers):

$$g_t + C_t = \frac{W_t N_t}{P_t} + \Omega_t. \tag{27}$$

The government in the economy is responsible for fiscal policy. Due to assumption that government expenditures are fixed ($g_t = g > 0$) the main fiscal instrument is lump-sum taxes. It is expressed by Taylor rule. This rule defines the level of lump-sum taxes τ_r at each period of time depending on the government liabilities deviation from their targeted level at this period $b_t - b$:

$$\tau_t = \tau + f^F \left(b_t - b\right), \tag{28}$$

where τ_t reflects the target level of lump-sum taxes, f^F is a Taylor coefficient. Choosing the value of Taylor coefficient the government can implement active (if $f^F < r$) or passive (if $f^F > r$) fiscal policy. Here r is a target level of real interest rate.

2.4 Central bank and monetary policy

Central bank in the economy is responsible for monetary policy. It implements Taylor rule as well. The monetary policy rule defines real interest rate r_t depending on the deviation of inflation from the target level $\pi_t - \pi$:

$$r_t = r + f^M \left(\pi_t - \pi\right), \tag{29}$$

where r is the target real interest rate, f^M is a Taylor coefficient. Supposing target level of inflation equals zero ($\pi = 0$), active monetary policy means real interest rate rises in the current rate of inflation $\left(f^M > 0\right)$. Passive monetary policy means real interest rate falls in the current rate of inflation $\left(f^M < 0\right)$.

2.5 Fiscal and monetary policies' regimes

Table 1 summarizes conditions that determine passive and active policies' regimes. They follow the logic presented in [32].

Table 1: Monetary and fiscal policies' regimes

	Monetary policy	**Fiscal policy,**
Active	$f^M > 0$	$f^F < r$
Passive	$f^M < 0$	$f^F > r$

Economically speaking, active monetary policy regime tolerates inflation increase in case of positive deviation of inflation from its target level. Consequently, passive monetary policy regime sustains inflation rising in case of positive deviation from the target level.

For fiscal policy regimes the logic is the opposite. The regime of fiscal policy is called passive when the government stabilizes debt by generating sufficient tax revenues. Active fiscal policy means that the government disregards budget constraint and allows debt rising without collecting more taxes in order to finance increasing debt.

2.6 Dynamical system representing the model

As a result of solving consumer's and producer's optimization problem and given government debt dynamics we obtain the following model:

$$\begin{cases} \dot{C}_t = (r_t - \rho)\, C_t - \xi\,(\xi + \rho)\, b_t, \\ \dot{b}_t = r_t b_t + g_t - \tau_t, \\ \dot{\pi}_t = \left(\rho + \xi\,(\xi + \rho)\,\frac{b_t}{C_t}\right)\pi_t - \frac{\varepsilon - 1}{\theta}\left(C_t^{1+\varphi} - 1\right). \end{cases} \tag{30}$$

Taking into consideration rules that reflect fiscal (28) and monetary (29) policies, and $\pi = 0$, system (30) takes the following main system of differential equations presenting dynamics of consumption, government bonds and inflation:

$$\left\{ \begin{array}{l} \dot{C}_t = (r-\rho)\, C_t + f^M \pi_t C_t - \xi\,(\xi+\rho)\, b_t, \\ \dot{b}_t = \left(r - f^F\right) b_t + f^F \pi_t b_t + g - \tau + f^F b, \\ \dot{\pi}_t = \left(\rho + \xi\,(\xi+\rho)\, \frac{b_t}{C_t}\right) \pi_t - \frac{\varepsilon-1}{\theta} \left(C_t^{1+\varphi} - 1\right). \end{array} \right. \tag{31}$$

3 Equilibrium dynamics

In this Section, we analyze system (31) from the side of local equilibrium determinacy and indeterminacy, including global indeterminacy, with respect to different monetary and fiscal policies regimes.

3.1 Steady states

System (31) possesses several steady states depend on values of model parameters. First of all, we focus on the plausible steady state levels of consumption, bonds and inflation as follows:

$$C^{ss} = 1,\ b^{ss} = \frac{\tau - g}{r},\ \pi^{ss} = 0. \tag{32}$$

However, if the government were to pay a gross subsidy different than $R = \frac{\varepsilon}{\varepsilon-1}$, then the steady state value of consumption would depend on both ε and R.

3.2 Analysis of local equilibrium determinacy and indeterminacy

According to [11], local equilibrium determinacy is ensured in case the number of Jacobian matrix eigenvalues with positive real parts equals the number of "jump" variables. System (31) has two "jump" variables, consumption and inflation, and one "non-jump" variable, bonds. In economics "jumping" behaviour of variables indicates that these variables could be set at the time of the unanticipated shock in order to bring the economy back on the stable path. Consumption and inflation in system (31) are such variables. However, government bonds are predetermined and they cannot be changed significantly once unanticipated shock occurs.

Following [5, 30, 50], we investigate the conditions for system (31) local equilibrium determinacy with respect to different policies regimes.

For the Jacobian matrix of system (31) at the steady state point $(C^{ss}, b^{ss}, \pi^{ss})$

$$J_0^{ss} = \begin{pmatrix} r-\rho & -\xi\left(\xi+\rho\right) & f^M \\ 0 & r-f^F & f^M b^{ss} \\ -\frac{(\varepsilon-1)(1+\varphi)}{\theta} & 0 & \rho+\xi\left(\xi+\rho\right)b^{ss} \end{pmatrix} \tag{33}$$

the characteristic polynomial $det(J_0^{ss}-\lambda I)$ has the following form

$$\chi\left(\lambda\right) = \lambda^3 + p_1^{ss}\lambda^2 + p_2^{ss}\lambda + p_3^{ss}, \tag{34}$$

where $p_i^{ss}\ (i=1,2,3)$ are the coefficients of characteristic polynomial (34).

It is known that $-p_3^{ss} = detJ_0^{ss}$ equals the product of the roots. A necessary condition for local equilibrium determinacy can be provided with $detJ_0^{ss} < 0$ or $p_3^{ss} > 0$. In this case it means that the signs of the roots of (34) may be "+ + –" or "– – –". Knowing that $-p_1^{ss} = TrJ_0^{ss}$ equals the sum of the roots, for ensuring local equilibrium determinacy it is sufficient matrix J_0^{ss} determinant be negative and trace be positive. As a result, finding areas of parameters that satisfy conditions $detJ_0^{ss} < 0$ and $TrJ_0^{ss} > 0$ (or $p_3^{ss} > 0$ and $p_1^{ss} < 0$) we conclude about system's local equilibrium determinacy.

The coefficients of characteristic polynomial take the following forms:

$$\begin{aligned} p_1^{ss} &= f^F - 2r - \left(b^{ss}\xi\left(\rho+\xi\right)\right), \\ p_2^{ss} &= \left(r-f^F\right)\left(r-\rho\right) + \left(2r-f^F-\rho\right)\left(b^{ss}\xi\left(\rho+\xi\right)+\rho\right) + \frac{\left(\varepsilon-1\right)\left(1+\varphi\right)f^M}{\theta}, \\ p_3^{ss} &= \left(b^{ss}\xi\left(\rho+\xi\right)+\rho\right)\left(f^F-r\right)\left(r-\rho\right) - \frac{\left(\varepsilon-1\right)\left(1+\varphi\right)f^M\left(b^{ss}\xi\left(\rho+\xi\right)-f^F+r\right)}{\theta}. \end{aligned} \tag{35}$$

Analyzing conditions for p_1^{ss} and p_3^{ss} with respect to different policies regimes, we formulate the following Proposition.

Proposition 1 *(Monetary and fiscal dynamics in case non-Ricardian consumers: $\xi > 0$). Consider the dynamical system (31). Let $q = \frac{(b^{ss}\xi(\rho+\xi)+\rho)\left(r-f^F\right)(r-\rho)\theta}{(\varepsilon-1)(1+\varphi)(b^{ss}\xi(\rho+\xi)-f^F+r)}$, $l = \xi\left(\xi+\rho\right)b^{ss}$; $\rho,\xi,\varphi,\theta,\tau$, and g are the positive parameters such that $r = \rho$, $\varepsilon > 1$, and $\tau > g$.*

(1) Suppose that at steady state (32), monetary and fiscal policies are active, then dynamics are never locally determinate.

(2) Suppose that at steady state (32), monetary and fiscal policies are passive. If $f^F \in (r; r+l)$, then dynamics are always locally determinate. If $f^F \in (r + l; 2r+l)$, then dynamics are locally determinate only if $f^M > -q$. If $f^M < -q$ and $f^F > 2r+l$, then dynamics are locally indeterminate of degree 1.

(3) Suppose that at steady state (32), *monetary policy is active and fiscal policy is passive. If* $f^F \in (r+l; 2r+l)$, *then dynamics are always locally determinate. If* $f^F \in (r; r+l)$, *then dynamics are locally determinate only if* $f^M < |q|$.

(4) Suppose that at steady state (32), *monetary policy is passive and fiscal policy is active. If* $f^M < -q$, *then dynamics are locally determinate.*

Proof. We investigate conditions on $detJ_0^{ss}$ and TrJ_0^{ss} by the coefficients of characteristic polynomial (34): p_1^{ss} and p_3^{ss}. In case monetary and fiscal policies are active ($f^M > 0, f^F < r$) the condition $detJ_0^{ss} > 0$ (or $p_3^{ss} < 0$) holds. That means active monetary and fiscal policies can never imply one negative and two positive eigenvalues of Jacobian matrix (33) and, therefore, ensure local equilibrium determinacy.

When monetary policy is now passive but fiscal policy at steady state $(C^{ss}, b^{ss}, \pi^{ss})$ is still active ($f^M < 0, f^F < r$), $detJ_0^{ss} < 0$ only if $f^M < -q$. Knowing that active fiscal policy implies $TrJ_0^{ss} > 0$, local equilibrium determinacy is achieved while monetary policy is "more passive" ($f^M < -q$).

Let monetary and fiscal policies are passive ($f^M < 0, f^F > r$). If $f^F \in (r; r+l)$, then all conditions are satisfied ($detJ_0^{ss} < 0$ and $TrJ_0^{ss} > 0$), implying one negative and two positive roots. That means local dynamics are always determinate. When $f^F \in (r+l; 2r+l)$ for obtaining $detJ_0^{ss} < 0$ it is necessary $f^M > -q$. In other words, dynamics are locally determinate only if monetary policy is not "very passive". Moreover, it is known that dynamics are locally indeterminate of degree 1 if Jacobian matrix J_o^{ss} of (33) has two negative and one positive roots that can be investigated if $detJ_0^{ss} > 0$ and $TrJ_0^{ss} < 0$. If $f^M < -q$ and $f^F > 2r+l$, then $detJ_0^{ss} > 0$ and $TrJ_0^{ss} < 0$ are satisfied and there exist two negative and one positive roots, and local indeterminacy of degree 1.

Assume monetary policy is now active but fiscal policy is still passive ($f^M > 0, f^F > r$). If $f^F \in (r+l; 2r+l)$ all conditions are satisfied ($detJ_0^{ss} < 0$ and $TrJ_0^{ss} > 0$), implying one negative and two positive roots. That means local dynamics are always determinate. When $f^F \in (r; r+l)$ for obtaining $detJ_0^{ss} < 0$ it is necessary $f^M < |q|$. In other words, dynamics are locally determinate only if monetary policy is not "very active". □

In special case when both the determinant $detJ_0^{ss} > 0$ and trace $TrJ_0^{ss} > 0$ of Jacobian matrix (33) are positive (i.e. $p_1^{ss} < 0$ and $p_3^{ss} < 0$) there are three positive roots of (34) ("$+++$"). This case must therefore be associated with a stable manifold of dimension zero (an unstable steady state) and existence of explosive solution. Trajectories that diverge from the steady state may converge to a limit cycle or a more complicated attracting set, or diverge to infinity. This means that local analysis of the properties of the Jacobian matrix evaluated at the steady state cannot be used to determine the eventual fate of the trajectories, i.e. using linear approximation of system (31). Under those circumstances, we might obtain global indeterminacy, if an attracting set appears, for the following values of policy parameters: $f^F < 2r + l$ and $f^M > -g$.

3.3 Conditions for equilibrium determinacy in Ricardian economy ($\xi = 0$)

One of the key features of the presented model is possibility to consider system (31) in the limit of infinitely lived consumers (assuming economy with Ricardian consumers). This is achieved by supposing $\xi = 0$. In this way, consumption dynamics are not influenced by government debt due to the fact government bonds are not considered as net wealth. Moreover, government bond dynamics do not affect inflation dynamics. Supposing economy with infinitely lived consumers, we provide analysis of local equilibrium determinacy with respect to different policies regimes as well.

Jacobian matrix (33) of model (31) with Ricardian consumers evaluated at the steady state point $(C^{ss}, b^{ss}, \pi^{ss})$ is expressed as follows:

$$\tilde{J}_0^{ss} = \begin{pmatrix} r-\rho & 0 & f^M \\ 0 & r-f^F & f^M b^{ss} \\ -\frac{(\varepsilon-1)(1+\varphi)}{\theta} & 0 & \rho \end{pmatrix} \tag{36}$$

The characteristic polynomial of system (31) in case Ricardian consumers can be written as

$$\tilde{\chi}\left(\tilde{\lambda}\right) = |\tilde{J}_0^{ss} - \tilde{\lambda} I| = \left(r - f^F - \tilde{\lambda}\right) V\left(\tilde{\lambda}\right), \tag{37}$$

where

$$\begin{aligned} &V\left(\tilde{\lambda}\right) = \tilde{\lambda}^2 + v_1\tilde{\lambda} + v_2, \\ &v_1 = -r, \\ &v_2 = \rho\,(r-\rho) + \frac{f^M\,(\varepsilon-1)\,(\varphi+1)}{\theta}. \end{aligned} \tag{38}$$

One of the roots of (37) is $r - f^F$. The other two roots are determined by the $V\left(\tilde{\lambda}\right)$. Knowing that $-v_1$ equals the sum of the roots and v_2 equals the product of the roots we can define the sign of the real parts of the roots.

Further, as postulated in [30] in Ricardian economy steady state is characterized by modified golden rule that implies $r = \rho$. As a result, the relations between coefficients of characteristic polynomial (38) and determinate and trace of submatrix of Jacobian matrix (36) are following:

$$\begin{aligned} \widetilde{Tr}\tilde{J}_0^{ss} &= -v_1 = r, \\ \widetilde{det}\tilde{J}_0^{ss} &= v_2 = \frac{f^M(\varepsilon - 1)(\varphi + 1)}{\theta}. \end{aligned} \tag{39}$$

Similar to system with non-Ricardian consumers the equilibrium can be locally determinate only if the number of positive real parts of eigenvalues equals the number of "jump" variables. After analyzing conditions with respect to different policies regimes we formulate the following Proposition.

Proposition 2 *(Monetary and fiscal dynamics in case Ricardian consumers: $\xi = 0$). Consider the dynamical system* (31) *with Jacobian matrix* (36). *Let $\rho, \xi, \varphi, \theta, \tau$, and g are the positive parameters such that $r = \rho$, $\varepsilon > 1$, and $\tau > g$.*

(1) Suppose that at steady state (32)*, monetary policy is passive and fiscal policy is active or, alternatively, monetary policy is active and fiscal policy is passive. Then, dynamics are always locally determinate.*

(2) Suppose that at steady state (32)*, monetary and fiscal policies are active. Then, dynamics are locally indeterminate of degree 1.*

(3) Suppose that at steady state (32)*, monetary and fiscal policies are passive. Then, dynamics are never locally determinate.*

Proof. We investigate conditions on $det\tilde{J}_0^{ss}$ and $Tr\tilde{J}_0^{ss}$ by identifying the sign of the root $\tilde{\lambda}_1 = r - f^F$ and the coefficients of characteristic polynomial (38) v_1 and v_2. In case fiscal policy is passive $(f^F > r)$ the sign of the root $\tilde{\lambda}_1$ is negative. If monetary policy is active $(f^M > 0)$, then $\widetilde{det}\tilde{J}_0^{ss} > 0$, and $\widetilde{Tr}\tilde{J}_0^{ss} > 0$ implies two positive roots $(\tilde{\lambda}_2, \tilde{\lambda}_3)$. That means local dynamics are always determinate. If fiscal policy is now active $(f^F < r)$ and monetary policy is still active $(f^M > 0)$, then the sign of the root $\tilde{\lambda}_1$ is positive. Hence, dynamics are never locally determinate because the Jacobian matrix $\tilde{J}_0^{ss}$ has three positive roots.

If monetary policy is passive $(f^M < 0)$, then $\widetilde{det}\tilde{J}_0^{ss} < 0$, and $\widetilde{Tr}\tilde{J}_0^{ss} > 0$ implies one positive and one negative roots $(\tilde{\lambda}_2, \tilde{\lambda}_3)$. If fiscal policy is active, then the sign of the root $\tilde{\lambda}_1$ is positive. And, dynamics are always locally determinate. If fiscal policy is passive the sign of the root $\tilde{\lambda}_1$ is negative then Jacobian matrix $\tilde{J}_0^{ss}$ has two negative roots and one positive root. Therefore, dynamics are locally indeterminate of degree 1. □

Consequently, assuming nonzero probability of death leads to consider a debt as a net asset by consumers. This affects their behavior and creates an additional channel for fiscal and monetary policy interaction, resulting in changing conditions for local equilibrium determinacy as well.

3.4 Calibration and graphical analysis

We calibrate the economy as follows. Let the time be a year. Following [38, 40, 41, 48] the quarterly discount factor β in discrete time models is set at 0.99. Knowing that $\rho = 1-\beta$, we find out $\rho = 0.01$ per quarter and $\rho = 0.04$ per year. Assuming non-Ricardian consumers are more impatient than Ricardian consumers or $r > \rho$ as in [30], we set target level of real interest rate at 0.0401. The chosen value of real interest rate is close to real interest rate postulated in [5, 6, 41].

In [41] the probability of survival is set at 0.995 to match an average life expectancy of 50 years. It consistent with the average age of the population in developed countries. Thereby, we set the value of death probability ξ is equal to 0.005.

The parameter of monopoly power ε should be set at value more than 1 in case of reflecting the economy with monopolistic competition. Moreover, assuming $\varepsilon \to \infty$, markup of price over marginal cost tends to unity, $\frac{\varepsilon}{\varepsilon-1} \to 1$ and reflecting economy in absence of monopoly power. In the literature (see, e.g. [38, 40, 41, 51]), the value ε varies from 6 to 21. We set the following parameter at 10 that makes markup of price over marginal cost be equal around 1.11.

Following [24], we choose the value of Rotemberg adjustment costs. It is shown in [24] that New Keynesian Phillips curves derived by using Rotemberg and Calvo price setting approaches have the same structure. Moreover, the authors provide results that the realistic value of Rotemberg's price adjustment costs depends on the steady state markup of price over marginal cost ($\frac{\varepsilon}{\varepsilon-1}$ in our case) and the probability of firms' price optimization that have to be chosen while considering Calvo price-setting mechanism. It is demonstrated in the paper that for steady state markup equals to 1.1 the range of Rotemberg's adjustment costs varies from around 4 to 192, depending on the percent of reoptimizing firms. We have chosen the value $\theta = 100$ that indicates Rotemberg adjustment costs consistent with approximately 25% of reoptimizing firms.

Assuming $C^{ss} = 1$, we derive he following condition on parameters: $\tau - g = \frac{r(r-\rho)}{\xi(\xi+\rho)}$. Choosing r, ξ, and ρ as above we find out that $\tau - g = 0.018$ and $b^{ss} = 0.444$. Then we have chosen the value of the government spending that

$g > \frac{C^{ss}}{\varepsilon}$. We assume government spending equals approximately 15% of output or $g = 0.15$ and define $\tau = 0.168$.

Values of parameter referring to the inverse of the labor supply elasticity φ in literature vary greatly. Following [38] or [40] we have chosen this parameter to be $\frac{1}{2}$.

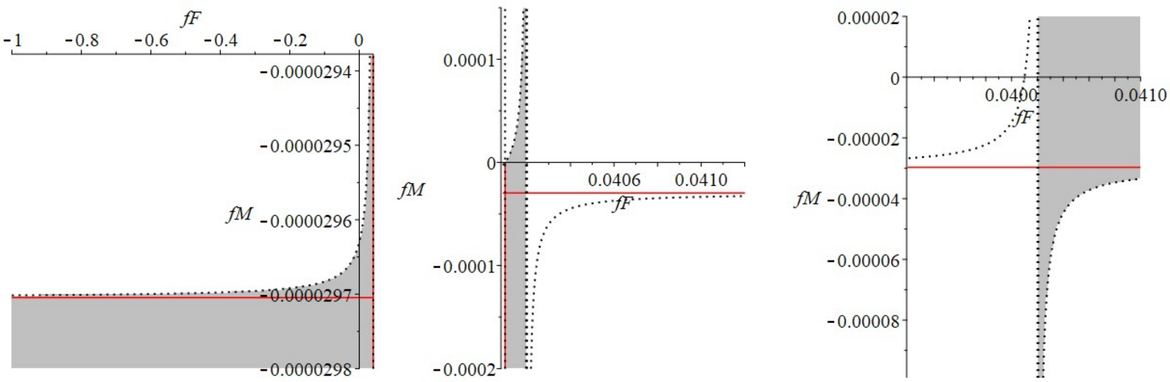
In table 2 all chosen values of parameters for model (31) are summarized.

Table 2: Calibration values of the parameter model

Parameter	**Description**	**Value**
f^M	Taylor coefficient of monetary policy rule	the parameter of interest
f^F	Taylor coefficient of fiscal policy rule	the parameter of interest
r	Target level of interest rate	0.0401
τ	Target level of lump-sum tax	0.168
ξ	Probability of death	0.005
θ	Adjustment costs	100
ρ	Time preferences	0.04
ε	Elasticity of substitution for the heterogeneous products	10
φ	Inverse of the labor supply elasticity	$\frac{1}{2}$
g	Government spending	0.15

Figures 1 and 2 prove Proposition 1 in accordance with chosen values of parameters of system (31). Areas shaded gray in Figure 1 correspond to areas where the necessary condition for local equilibrium determinacy is satisfied ($detJ_0^{ss} < 0$). Combining them with gray area in Figure 2 we obtain sufficient condition for local determinacy ($detJ_0^{ss} < 0$ and $TrJ_0^{ss} > 0$). It is necessary to point out that gray area in 2 intercepts all panels in 1 that allows make conclusion about local determinacy by analyzing only panels in the Figure 1. For example, in Figure 1(a) it is shown that in case passive monetary policy ($f^M < 0$) and active fiscal policy (assuming $f^F = -0.2$) local equilibrium determinacy is ensured if $f^M < -q$ or $f^M < -0.0000297$. That corresponds to point (4) of Proposition 1. Figure 1(b) demonstrates that if both policies are passive and $f^F \in (r; r + l)$ (supposing $f^F = 0.04018$), then dynamics are always determinate. That corresponds to point (2) of Proposition 1. Figure 1(c) illustrates that if monetary policy is active and fiscal policy is passive and $f^F \in (r + l; 2r + l)$ (for instance, $f^F = 0.0405$) dynamics are always determinate. That corresponds to point (3) of Proposition 1.

Figure 3 proves point (2) of Proposition 1 and displays that dynamics are indeterminate of degree 1 when $f^M < -q$ and $f^F > 2r + l$. For example, if $f^F = 0.1$ and $f^M = -0.00003$, then $detJ_0^{ss} > 0$, $TrJ_0^{ss} < 0$ and indeterminacy of



(a) $f^F < r$ (b) $f^F \in (r; r+l)$ (c) $f^F > r+l$

Figure 1: The satisfaction area of the necessary condition for local equilibrium determinacy: $detJ_0^{ss} < 0$ or $p_3 > 0$ (red horizontal line is an asymptote of function p_3; red vertical line demonstrates value of parameter $r = 0.0401$).

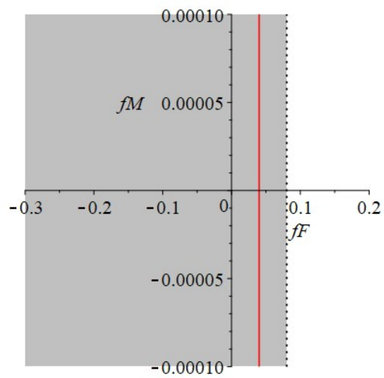


Figure 2: The satisfaction area of the condition for local equilibrium determinacy: $TrJ_0^{ss} > 0$ or $p_1 < 0$ (red vertical line demonstrates value of parameter $r = 0.0401$).

degree 1 occurs.

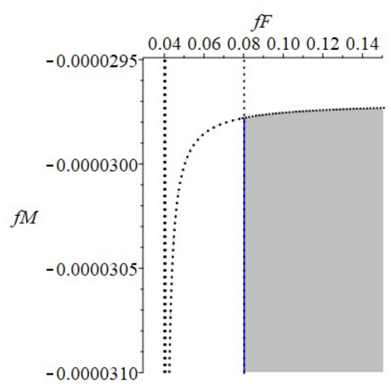


Figure 3: The satisfaction area of the condition for local equilibrium indeterminacy of degree 1: $detJ_0^{ss} > 0$, $TrJ_0^{ss} < 0$ (blue vertical line demonstrates value of parameter $2r + l = 0.080302$).

Figure 4 shows that if $f^F < 2r+l$ and $f^M > -q$ (in particular, $f^F = 0.01$ and

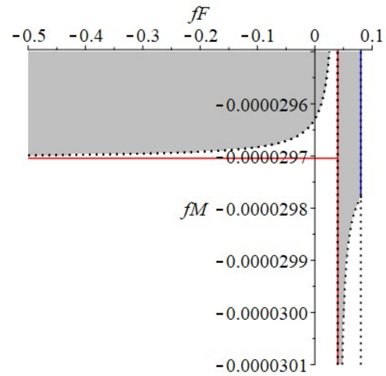
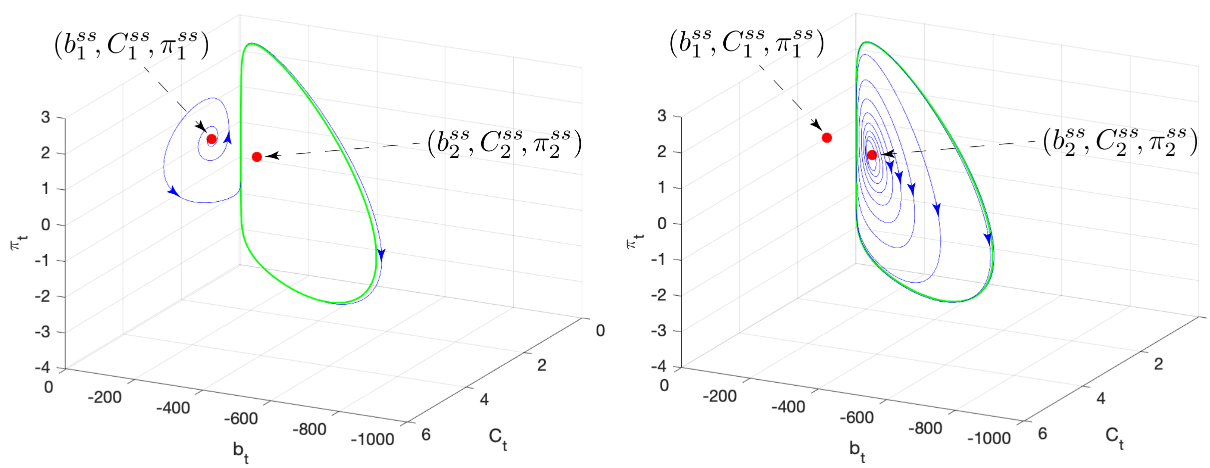


Figure 4: Regions in the $(f^F,\ f^M)$ space where the explosive equilibrium obtains: $detJ_0^{ss} > 0$, $TrJ_0^{ss} > 0$ (red and blue vertical lines demonstrate values of parameters $r = 0.0401$ and $2r + l = 0.080302$, respectively; red horizontal line is an asymptote of function p_3^{ss}).

$f^M = 0.1$, i.e., the both policies are active) then the model demonstrates explosive local dynamics and attracting equilibrium cycle exists. Areas shaded gray in Figure 4 correspond to regions where the condition for global indeterminacy is satisfied: $detJ_0^{ss} > 0$ and $TrJ_0^{ss} > 0$.



(a) for steady state $(C_1^{ss}, b_1^{ss}, \pi_1^{ss})$ (b) for steady state $(C_2^{ss}, b_2^{ss}, \pi_2^{ss})$

Figure 5: The stable limit cycle (green trajectory) for steady state points $(C_1^{ss}, b_1^{ss}, \pi_1^{ss}) = (1, 0.444, 0)$ (a) and $(C_2^{ss}, b_2^{ss}, \pi_2^{ss}) = (0.977773,\ -130.81334,\ -0.30203)$ (b), respectively.

Figure 5 shows a stable limit cycle which could be localized from some neighbourhoods of the unstable steady state points $(C_1^{ss}, b_1^{ss}, \pi_1^{ss}) = (1, 0.444, 0)$ (Fig. 5(a)) and $(C_2^{ss}, b_2^{ss}, \pi_2^{ss}) = (0.977773,\ -130.81334,\ -0.30203)$ (Fig. 5(b)), respectively. In this case, model (31) possesses explosive local dynamics as well as globally indeterminate behavior – a stable limit cycle. Along this cycle, all variables perpetually oscillate in an endogenous, deterministic fashion. Hence, unlike linear ones, non-linear business cycle models can generate sustained ocsilations even in the absence of exogenous shocks (for instance, via limit cycles or chaos).

4 Future challenges

The study of business cycles via NK models is a vibrant area of macroeconomic research that faces many open challenges. NK models can exhibit undesired irregular and cyclical dynamics at plausible settings of parameters policy that leads to implications, ranging from long-term unpredictability to global indeterminacy. The existence of attracting cycles represents a severe case of policy-induced macroeconomic instability. The occurrence of global indeterminacy is consistent with the fact that economists who provide short term forecasts are rarely willing to provide long term forecasts [3]. Nevertheless, discovering and tracking stable, unstable, deterministic chaotic and stochastic processes which might observe in models, allows to control and forecast their dynamics [1,2,4,20,26]. An important task in both qualitative and quantitative investigation of dynamical systems is the study of limiting behavior of a system after a transient process, i.e., the problem of revealing and analysis of all possible limiting oscillations caused by local bifurcations in the vicinity of the stationary set (trivial boundary of the global stability) or nonlocal bifurcations (hidden boundary of the global stability), when violating the global stability boundary in the space of parameters. In the latter case hidden attractor can be born [28,33]. Are business cycle fluctuations a response to a persistent exogenous shocks or do they reflect of a strong endogenous mechanism which produces recurrent boom-bust phenomena? Combining of analytical and numerical methods of mathematical control theory and macroeconomic analysis by examing the notion of stochastic limit cycles can clarify this matter during the next few years.

5 Conclusion

This paper derives the continuous time New Keynesian model and provides analysis of monetary and fiscal policies' interactions impact on local equilibrium determinacy. The model is constructed by applying structure of the economy presented in [30]. There are four main agents in the economy, namely representative consumer, representative producer, the government and Central bank. In contrast to [30], we incorporate Rotemberg price-setting mechanism instead of Calvo. Departing from Ricardian equivalence, we consider representative consumer faces constant probability of death. The model is constructed as a system of three differential equations presenting dynamics of consumption, government bonds and inflation. The system incorporates 10 parameters, including f^F and f^M that indicate Taylor coefficients of fiscal and monetary policies' rules respectively.

The main findings of the paper are formulated in Proposition 1. Our analysis proves the following results. First, local equilibrium determinacy cannot be ensured in case of active monetary and fiscal policies. In other words, if fiscal authority does not increase taxes collection in response to increasing government debt, but monetary policy rule is aimed at stabilizing inflation, then the uniqueness of the equilibrium solution is never achieved. Second, local equilibrium determinacy in any other policies' combinations depends on ranges of f^F and f^M. Local equilibrium determinacy can be achieved only through appropriate choosing values of these parameters. Finding appropriate values for f^M depends on a critical value q that is defined by system's parameters and the Taylor coefficient f^F. Moreover, the main results are compared with those that are derived for the case of Ricardian consumers ($\xi = 0$). It is shown that government debt is a net asset, there appears an additional channel for fiscal and monetary policy interaction which changes conditions for local equilibrium determinacy. In addition to determinate and indeterminate equlibria, a larger set of equilibria including those in which endogenous variables remain bounded but are never expected to return to the steady state, such as limit cycles, might appear, leading to global indeterminacy.

Acknowledgements

The work is supported by the 2020-2021 program Leading Scientific Schools of the Russian Federation (project NSh-2624.2020.1).

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