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*Nonlinear oscillations*

# Synthesis Algorithm for Control System with Saturated Actuator

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**Absract.** The paper presents a generalized algorithm for designing closed optimal systems with a dynamic controller and a nonlinear actuator. The purpose of the proposed algorithm is to identify the area of system performance degradation, manifested as unintentional oscillations, due to the influence of actuator nonlinearity and reduce this area. The algorithm is implemented using a heuristic search algorithm and the method of sequential nonlinear correction. The results of the algorithm operation are shown on the PI control system of the angular plant motion with a saturated actuator. Simulation of the system with various input parameters showed the possibility of fluctuations in the system output. The numerical analysis results of the nonlinear system are presented in the form of exponential diagrams of the performance error, covering all input parameters.

**Key words:** heuristic search algorithm; controller tuning; optimization; nonlinearity compensation; numerical analysis; generalized sensitivity function

# 1 Introduction

Actuators are usually designed without large reserves of traction forces due to restrictions on the mass and size of the engineering object. During operation, the actuator is close to the limit of its traction, which can lead the system to the verge of stability. The system stability can be affected by external disturbances, large amplitudes of the input signal, and sharp manoeuvring. Therefore, the controller synthesis based on ensuring global asymptotic stability provides the desired result only in a limited range of system parameters [1].

Studies show that nonlinearities such as saturation in terms of speed and level of the control signal have a valuable negative impact on the operation of systems [2–4]. In the literature, the so-called integrator excitation phenomenon in a feedback system, or windup, is known, when the control signal is limited, the integrator accumulates an error, leading to undesirable output oscillations [1–3]. The increasing amplitude of the output oscillations can lead to the destruction of the equipment. Hence, an additional saturation compensation block is introduced for the nominal regulator. The static anti-windup compensator is implemented as part of the standard controller in the form of an amplified mismatch between the input and output of the saturation. That is, its structure involves measuring the saturation level. However, such an approach is unrealizable if a human operator acts as a regulator. Another way to compensate for saturation is to introduce a phase-advancing nonlinear filter into the rive control loop, which increases the phase stability margin [5–7]. The phase compensation method has found application in aircraft control systems to avoid oscillations about the axes of inertia. Also, a promising approach can be nonlinearity compensation using neural networks, as shown in [8], where the dependence of the material size on temperature has the form of hysteresis and negatively affects the positioning accuracy of the drive. It is necessary to focus on the safety of nonlinear electric drive systems, intensive research on stability and the development of safe controls due to the spread of various robotic technology in human life. An illustrative example of this fact is the existence of hidden attractors in the system, the identification of which is possible only with the help of specially developed computational procedures [9, 10].

The authors have investigated the ability of oscillation prevention in the manual control system of an aircraft and a robotic arm [6, 11]. These works show the effective use of a serial nonlinear corrective device to stabilize the controlled parameter. It seems that for all systems, there is a range of oper-

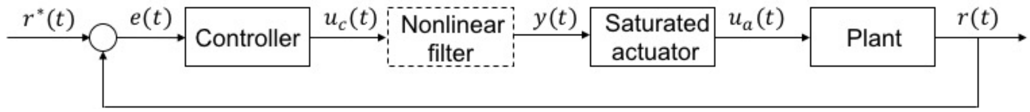


Figure 1: Control loop structure

ating parameters and a saturation level that limits this range. It has led to an assumption that it is possible to select numerically the area of acceptable operating parameters for a particular system. This paper aims to develop a technique that allows any control system to mistakenly identify the field of input signal parameters in which stability loss is possible.

# 2 Optimal Control System with Saturated Actuator

## 2.1 Brief description of the synthesis algorithm

Some works show that it is possible to design a controller in a system with a constraint by optimal control methods. However, such solutions lead to relay control, significant computational difficulties, and, ultimately, failures in implementation in digital systems [1, 12]. The approach proposed in this paper is also based on the principle of system optimality. The algorithm contains additional steps associated with a nonlinear correction method described below. Let us assume that the block diagram of the system control corresponds to that shown in Fig. 1, where a nonlinear correction device (or nonlinear filter) is already embedded into the actuator speed control loop.

**Algorithm 1** My algorithm

1: Find the parameters of the nonlinear filter corresponding to the maximum phase margin of the system.
2: Find the parameters of the controller.
3: Conduct a numerical system analysis for various input system parameters.

## 2.2 Explanations of Algorithm 1

The nonlinear filter proposed for use is described by the following equations [6, 7, 13]:

$$y(t) = |x(t)| sign(\sigma(t)), \tag{1}$$
$$\sigma(t) = W_{ph}(j\omega)x(t), \tag{2}$$
$$W_{ph}(j\omega T) = \frac{T_1}{T}\frac{(Ts+1)}{(T_1 s+1)}, \tag{3}$$

Created advance by (3) is calculated as:

$$\alpha(\omega) = \arctan\frac{\omega T(1-\nu)}{1+\omega^2T^2\nu}, \tag{4}$$

where $\nu = T_1/T$.

The harmonic linearization coefficients of the nonlinear filter are determined by the well-known formulas [14, 15], and (1), (2), and (3) have the expressions:

$$a(\omega) = \frac{1}{\pi}[\pi - 2\alpha(\omega) + \sin(2\alpha(\omega)], \tag{5}$$
$$b(\omega) = \frac{1}{\pi}[1 - \cos(2\alpha(\omega))]. \tag{6}$$

Fig. 2 shows the amplitude and phase response of (1)-(3) under various $nu$.

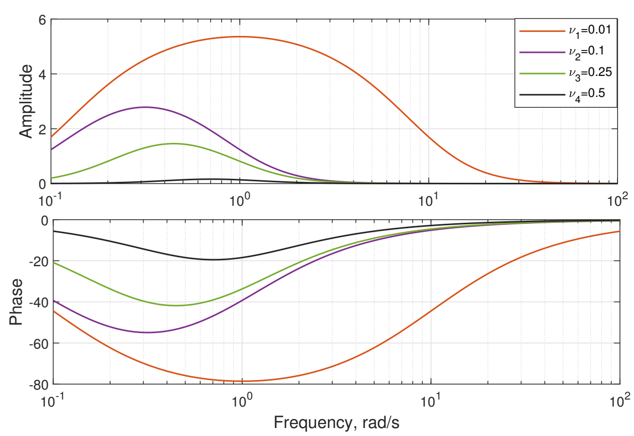


Figure 2: Frequency response of the nonlinear filter (1)-(3)

Fig. 2 shows the main used property of the nonlinear correction, which consists of the ability to control the phase response regardless of the magnitude of the input signal amplitude, which is important for nonlinear systems excited by signals with various amplitudes and frequencies.

In the second point of Algorithm 1, it is required to solve the optimization problem, which can be formulated as:

$$\max J(K) = \omega_c, \quad \text{subject to} \;\; g(K) < b, \tag{7}$$

where $\omega_c$ is the crossover frequency, $K$ is the vector of controller gains, and $g$ is the function limited by the parameter $b$. It is convenient to impose restrictions on the frequency indicators of the system quality, for example, the oscillation index. Since the cutoff frequency is approximately inversely proportional to the transient time, then (7) can be written as $\min J(K) = t_t$.

Solution (7) reduces to a standard linear programming problem, which in the general case can be solved by a gradientless search method. Many different efficiency search algorithms can be used in the best way for problems in consideration [16]. In this paper, the standard Nelder-Mead algorithm in MATLAB is used for simplicity. The result of solving the problem (7) is a set of coefficients of the controller.

In the last step of Algorithm 1, it is proposed to determine the area of acceptable input signals $r^*$ for the system under consideration and the found controller parameters. In this paper, it is proposed to construct the generalized sensitivity function as a function of input signal parameters. To calculate it, the assumption is introduced that the considered nonlinear system is uniformly convergent for the class of signals $r^*(t) = A\sin(\omega t)$ and for each $r^*(t)$ there is a unique steady-state periodic solution $y(t)$ such that $e(t) = r^*(t) - y(t)$ [17, 18]. By definition, the generalized sensitivity function for a nonlinear system is calculated as follows [18, 19]:

$$S(A,\omega) = \frac{||e(t)||_2}{||r^*(t)||_2}, \tag{8}$$

where $A$ and $\omega$ are the amplitude and frequency of the reference signal.

Equation (8) is the function of two arguments, which makes it possible to construct a three-dimensional diagram for greater clarity.

## 3 Illustrative Example

### 3.1 System Description

Let's execute the modelling of the feedback control system shown in Fig. 1 with a nominal controller, a nonlinear actuator, and a plant described below. The control law is described by a standard proportional-integral (PI) form aimed at eliminating the performance error $e(t) = r^*(t) - r(t)$. The transfer function of PI controller from performance error $e(t)$ to control signal $u_c(t)$ is written as [20]:

$$W_c(s) = K_p + \frac{K_i}{s}, \tag{9}$$

where $K_p$ and $K_i$ are the unknown gains to be found.

Let us assume that the control is performed by one variable - a coordinate or an angle. So, the transfer function of the plant together with the actuator from the control signal $u_c(t)$ to the system output $r(t)$ can be written as:

$$W_p(s) = \frac{k_1(k_2 s + 1)}{(T_a s + 1)(T_\theta s^2 + 2\xi\omega_\theta + \omega_\theta^2)s}, \tag{10}$$

where $T_a$ is the actuator time constant, and the rest of the parameters refer to the plant: $k_\theta$ is the gain, $T_\theta$ is the time constant, $\xi$ is the damping gain, and $\omega_\theta$ is the natural frequency.

The appearance of saturation is possible both in terms of the level and the speed of the signal. Consider speed saturation as the most problematic, which can be described by the function: $\mathrm{sat}_{\overline{u}_a} : F \longrightarrow [-\overline{u}_a, \overline{u}_a]$, where $\mathrm{sat}_{\overline{u}_a} = \mathrm{sign}(u_a)\min\{|u_a|, \overline{u}_a\}$, $\overline{u}_a > 0$.

## 3.2 Numerical Results

Numerical simulation is carried out with the following system parameters:

- initial parameters of the controller: $K_P^0 = 1$ and $K_i^0 = 1$;
- plant parameters: $T_a = 0.07$ s, $\overline{u}_a = 6$ deg/s, $K_1 = 128$, $k_2 = 0.23$, $\omega_\theta = 5$, $\xi = 0.7$;
- input is harmonic with amplitudes $A = [5...50]$ deg, and the frequencies $\omega = [0...20]$ rad/s.

To improve the stability and performance quality of the system, we use a nonlinear filter (1)-(4) and determine its time constants for a given control object and saturation level: $T = 0.01$ s and $T_1 = 50$ s.

Fig. 3 shows the time response of the nonlinear system under different input parameters. It can be seen that the amplitude and frequency of the input affect the performance error (figures on the left), and there are a several inputs at which the performance error tends to zero (figure on the top right). The introduction of a nonlinear filter stabilizes the output (bottom right figure).

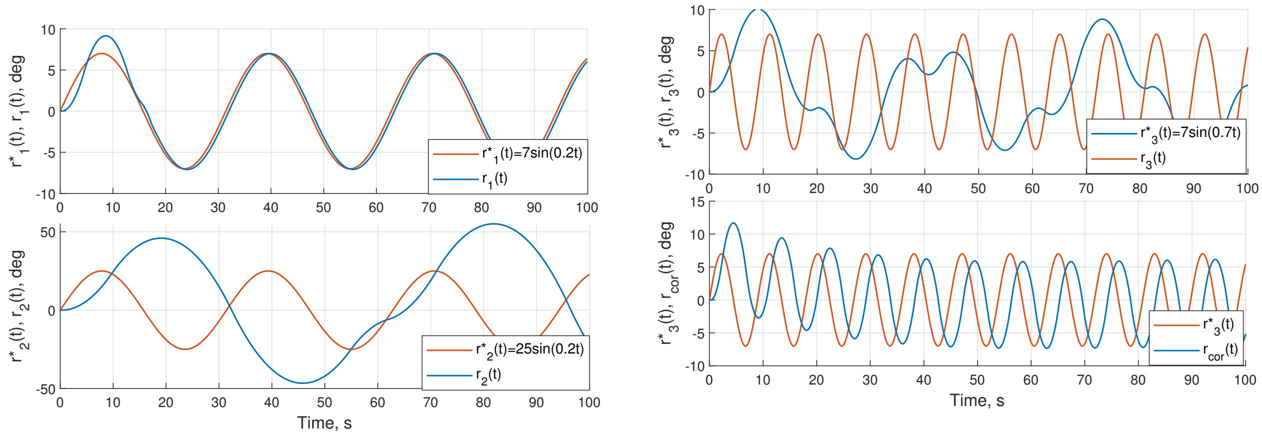


Figure 3: Time response of a nonlinear system for various input parameters; Time response of a system with nonlinear filter and without it

The search for PI controller parameters is presented in Fig. 4, which shows the controller coefficients $K_P$, $K_i$, the cutoff frequency $\omega_c$, and the achieved conditions imposed on the oscillation index $M < 1.25$ (7). As a result, the following coefficients $K_P = 128$, $K_i = 325$ are obtained.

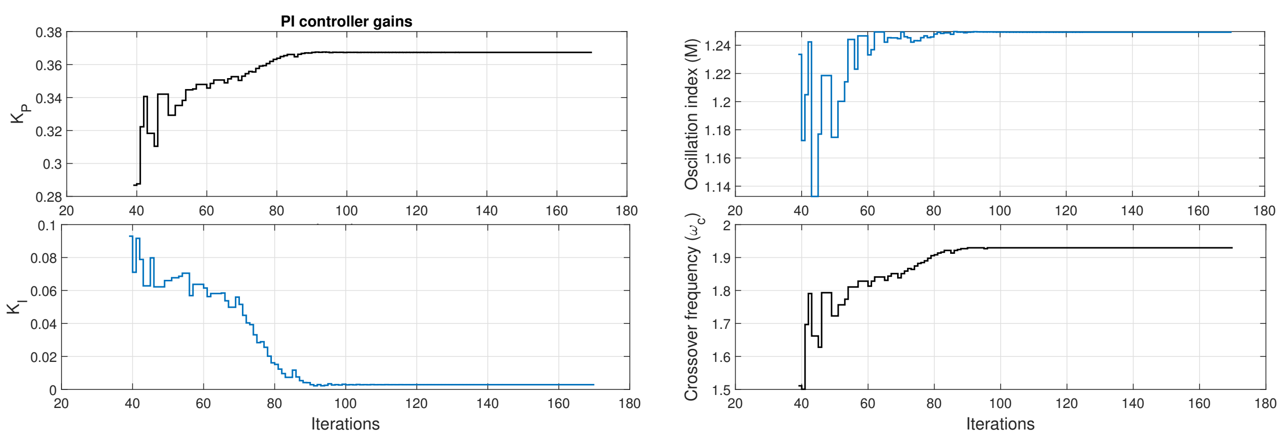


Figure 4: Illustration of the search algorithm

Fig. 5 shows diagrams of the generalized sensitivity function (8), reflecting the performance error in a nonlinear system without a nonlinear filter (a) and with a nonlinear filter (b). From these diagrams, you can determine at what input parameters the non-linearity has the greatest negative impact. It can also be seen that after the introduction of the nonlinear filter, the input parameter area expanded, that is, the performance error decreased by about a factor of two.

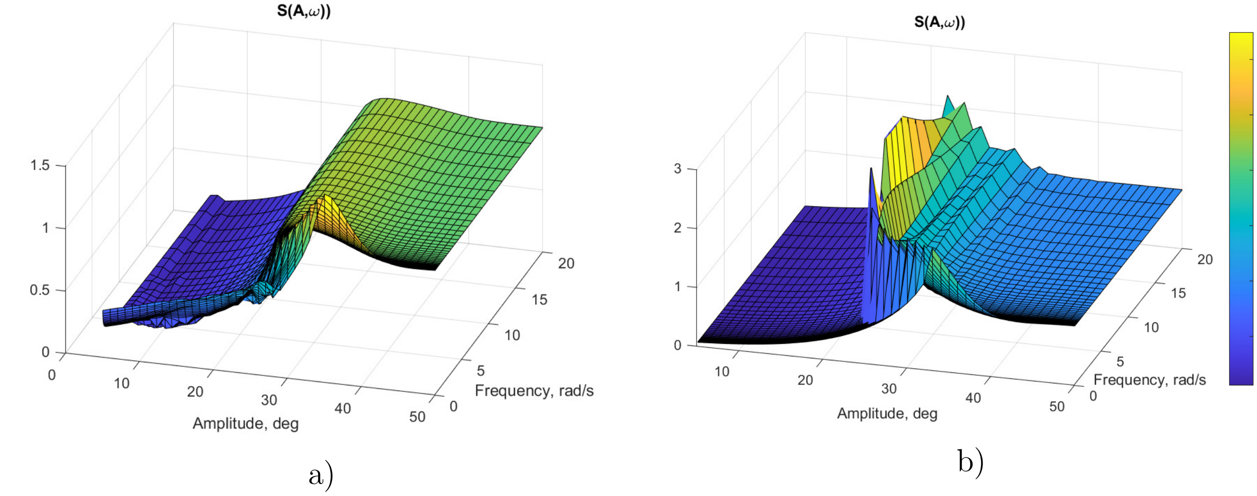


Figure 5: Performance error diagram based on generalized sensitivity function

## 4 Conclusion

The paper shows the dependence of nonlinear system performance on the input signals parameters and the effect of actuator saturation. An algorithm is proposed for synthesizing a control system with the nonlinear actuator. This algorithm consists of automatic tuning of the controller parameters, embedding a nonlinear corrective filter, as well as constructing the area of acceptable parameters of the system inputs that provide the required quality of control. The proposed algorithm can be applied in engineering control systems operating based on performance errors at the simulation stage and in real-time control. The results can be used in the design of systems and numerical analysis to identify the area of prohibited parameters for security purposes.

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