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Adaptive anti-synchronization between different hyperchaotic systems with uncertain parameters

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Abstract

This article deals with anti-synchronization between different hyperchaotic systems such as Lu system and Newton-Leipnik system; and Newton-Leipnik system and Rossler system using adaptive control method. Based on Lyapunov stability theory, the anti synchronization between a pair of hyperchaotic systems with fully unknown parameters are derived. An adaptive control law and a parameter update rule for unknown parameters are designed such that the hyperchaotic Newton-Leipnik system is controlled to be the hyperchaotic Lu system and hyperchaotic Rossler system is controlled to be the hyperchaotic Newton-Leipnik system. Numerical simulation results which are carried out using MATLAB, show that the adaptive control method is effective, easy to implement and reliable for anti-synchronizing of the considered hyperchaotic systems.

Keywords: Hyperchaos, Lu system, Newton-Leipnik system, Rossler system, Anti-synchronization, Adaptive control.

1. Introduction

In the past, modeling was restricted mainly to linear systems which would have been tackled using various analytical methods. But with the advent of powerful computers and related softwares, it is made possible to tackle nonlinear systems. Nonlinearity is the heart of dynamical systems is deterministic and changes with time. The applications of dynamical systems have nowadays spread to a wide spectrum of disciplines including science, engineering, biology, sociology etc. Study and analysis of nonlinear dynamics have gained immense popularity during the last few decades due to its important feature of any real-time dynamical system. In nonlinear systems, a small change in a parameter can lead to sudden and dramatic changes in both the qualitative and quantitative behavior of the system. Sometimes these may give rise to the complex behavior called chaos. In dynamical systems, the term chaos is applied to deterministic systems that are aperiodic and that exhibit sensitive dependence on initial conditions and parameter variations. Sensitivity means that a small change in the initial state will lead to progressively larger changes in later system states. The presence of chaos in physical systems has been extensively demonstrated and is very common. The main property of chaotic dynamics is its critical sensitivity to initial conditions, which is responsible for initially neighboring trajectories separating from each other exponentially in the course of time. The concept of chaos has been used to explain how systems that should be subject to known laws of physics may be predictable in the short term but are apparently random on a longer time scale.

A Hyperchaotic system is usually classified as a chaotic system with more than one positive Lyapunov exponent. Though the positive largest Lyapunov exponent does not indicate chaos as the negative largest Lyapunov exponent does not indicate stability [1], it is a fact that with the advent of computers and the increasing facility to perform a number of experiments for calculating characteristic exponents for the sake construction of various numerical characteristics attractors, the researchers working in dynamical systems consider that the instability of the solutions of the system occurs at this largest positive characteristic exponent [2]. Combine with one null exponent along the flow and one negative exponent to ensure the boundness of the solution. It means that hyperchaotic systems have more complex dynamical behaviours which can be used to improve the security of a chaotic communication system. Firstly, Hyperchaotic system introduced by Rossler [3] in 1979. Controlling synchronization of hyperchaotic system have attracted a great deal of attention from various fields and become a challenging work.

Since the idea of synchronizing chaotic systems was introduced by Pecora and Carroll [4] in 1990, they showed that it is possible to synchronize chaotic systems through a simple coupling. Synchronization of chaotic dynamical systems has been intensively studied by many researchers [5-8] and has attracted a great deal of interest in various field due to its important applications in ecological system [9], physical system [10], chemical system [11], modeling brain activity, system identification, pattern recognition phenomena and secure communications [12-14] etc.

In the recent years several different types of synchronization schemes have been proposed, such as time delay feedback approach [15], adaptive control [16-23], active control [24], back-stepping design method [25] and sliding mode control [26] etc, have been successfully applied to chaos synchronization. The concept of synchronization can be extended to generalized synchronization [27], complete synchronization [28], lag synchronization [29], phase synchronization [30], anti-synchronization [31], projective synchronization [32], modified projective synchronization [33] and hybrid synchronization [34] etc.

The control of chaotic systems is to design state feedback control laws that stabilizes the chaotic systems around the unstable equilibrium points. Adaptive control technique is used when the system parameters are unknown. In an adaptive method control law and a parameter update rule for unknown parameters are designed in such a way that the hyperchaotic response system is controlled to be the hyperchaotic master system. The idea of anti-synchronization is observed in periodic hyperchaotic systems, which is a phenomenon in which the state variables of synchronized systems with different initial values have the same absolute values but opposite signs. The sums of two signals are expected to converge to zero when anti-synchronization occurs.

Most of the studies in synchronization involve two identical /non-identical systems under the hypotheses that all the parameters of the master and slave systems are known a prior, a controller is constructed with the known parameters and systems are free from external perturbations. But in practical situations the uncertainties like parameter mismatch and external disturbances may destroy the synchronization and even break it. So it is necessary to design an adaptive controller and parameter update law for control and synchronization of chaotic systems consisting of unknown parameters to get rid of internal and external noises. From the literature survey, it is seen that with the development of nonlinear control theory, nowadays adaptive anti-synchronization method has become very much effective to control and synchronize of chaotic and hyperchaotic systems both with unknown parameters and external disturbances. This has motivated the authors to do significant study of the adaptive anti-synchronization between two pair of two hyperchaotic systems all having unknown parameters.

Recently, many authors have studied the control and synchronization for the chaotic and hyperchaotic systems. Zhang et al. [16] studied adaptive synchronization between two different chaotic systems with unknown parameters. Hu et al. [17] proposed adaptive control for anti-synchronization of Chua's chaotic system. In 2008, Salarieh and Shahrokhi [18] investigated adaptive synchronization of two different chaotic systems with time varying unknown parameters. In 2008, Wu et al. [19] proposed a scheme to synchronize hyperchaotic Chen system and generalized Henon-Heiles System of different structures via adaptive control. Zhang and Zhu [20] proposed anti-synchronization of two different hyperchaotic systems via active and adaptive control methods. Mossa et al. [21] investigated adaptive anti-synchronization of chaotic systems with fully unknown parameters in 2010. Mossa et al. [22] have investigated adaptive anti-synchronization of two identical and different hyperchaotic systems with uncertain parameters in 2010. In 2011, Li et al. [23] proposed Complete (anti-)synchronization of chaotic systems with fully uncertain parameters by adaptive control.

In this article, the authors have studied adaptive anti-synchronization between different hyperchaotic systems using adaptive control and parameter update rule. This article has been organized as follows. In Section 2, adaptive anti-synchronization method is discussed. In section 3, the system descriptions of Lu, Newton-Leipnik and Rossler systems are given. In Sections 4 and 5, adaptive anti-synchronization between Lu and Newton-Leipnik hyperchaotic systems; and Newton-Leipnik and Rossler hyperchaotic systems are discussed respectively. In Section 6, the conclusion of the work is presented.

2. Adaptive anti-synchronization

Consider the drive hyperchaotic system in the form of

$$\dot{x} = F(x) + f(x)\alpha, \tag{1}$$

where $x \in R^n$ is the state vector of the system, $\alpha \in R^m$ is the unknown parameter vector of the system, non linear term $F(x)$ is an $n \times 1$ matrix, $f(x)$ is an $n \times m$ matrix and the elements $f_{ij}(x)$ in the matrix $f(x)$ satisfy $f_{ij}(x) \in L_\infty$ for $x \in R^n$. On the other hand, the response system is assumed as

$$\dot{y} = G(x) + g(x)\beta + \mu(t), \tag{2}$$

where $y \in R^n$ is the state vector of the system, $\beta \in R^q$ is the unknown parameter vector of the system, non linear term $G(x)$ is an $n \times 1$ matrix, $g(x)$ is an $n \times q$ matrix, $\mu \in R^n$ is control input vector, and the elements $g_{ij}(x)$ in the matrix $g(x)$ satisfy $g_{ij}(x) \in L_\infty$ for $y \in R^n$.

Let $e = y + x$ be the anti-synchronization of the error dynamical system. The purpose of hyperchaos anti-synchronization is how to design the controller parameter $\mu(t)$, such that,

$$\lim_{t\to\infty} \|e(t)\| = \lim_{t\to\infty} \|y(t, y_0) + x(t, x_0)\| = 0,$$

where $\|.\|$ represents the Euclidean norm.

We add equation (2) to equation (1) and get

$$\dot{e} = F(x) + f(x)\alpha + G(x) + g(x)\beta + \mu(t), \tag{3}$$

The parameters belonging to the drive and the response systems are always unknown. Therefore, by using the adaptive control and the parameter update rule techniques, the adaptive nonlinear controller can be selected as

$$\mu = -F(x) - f(x)\hat{\alpha} - G(y) - g(y)\hat{\beta} - ke, \tag{4}$$

and adaptive laws of parameters are taken as

$$\dot{\hat{\alpha}} = [f(x)]^T e,$$
$$\dot{\hat{\beta}} = [g(y)]^T e, \tag{5}$$

then the response system (2) can anti-synchronize the drive system (1) globally and asymptotically, where k > 0 is a constant, $\hat{\alpha}$ and $\hat{\beta}$ are, respectively, estimations of the unknown parameters α and β, where α and β are constants.

Assume a positive Lyapunov function $V = \frac{1}{2}[e^T e + \overline{\alpha}^T \overline{\alpha} + \overline{\beta}^T \overline{\beta}]$, where $\overline{\alpha} = (\alpha - \hat{\alpha})$, $\overline{\beta} = (\beta - \hat{\beta})$.

With the choice of the adaptive control law and parameter update rule above for unknown parameters are designed, the time derivative of V along the solution in equation (3) will be smaller than zero. In other words, the error vector will approach to zero as time goes infinite and from Lyapunov stability theory [35], the states of the slave system and projected master system are asymptotically anti-synchronized.

3. Systems' descriptions

3.1 Lu system

The hyperchaotic Lu system [36] is described by

$$\begin{aligned} D_t x &= a(y-x) + w \\ D_t y &= -xz + cy \\ D_t z &= xy - bz \\ D_t w &= xz + dw, \end{aligned} \tag{6}$$

where x, y, z and w are state variables, and a, b, c and d are real constants. When $a = 36,\ b = 3,\ c = 20$ and d take different values, system performance different dynamics. When $-1.03 < d \le -0.46$, system has a periodic orbit; when $-0.46 < d \le -0.35$, system has chaotic attractor and when $-0.35 < d \le 1.30$, there are two index greater than zero system showing hyperchaotic attractor. The hyperchaotic attractors in $x-y-z$, $x-y-w$, $x-z-w$ and $y-z-w$ spaces are depicted through Fig. 1.

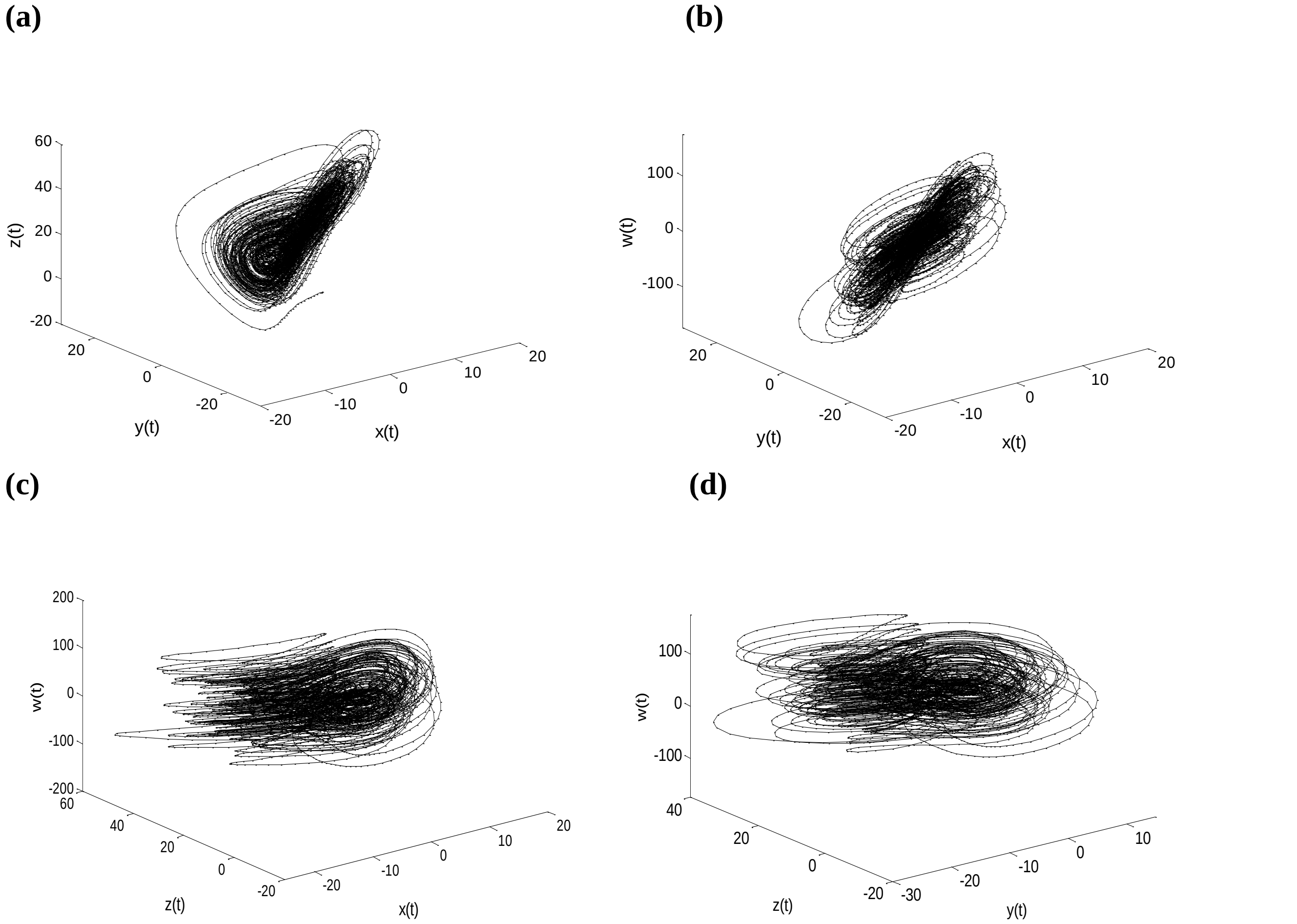


Fig.1. Phase portraits of hyperchaotic Lu attractor in (a) *x*-*y*-*z* space *(b) x*-*y*-*w* space (c) x-*z*-*w* space and (d) y-*z*-*w* space.

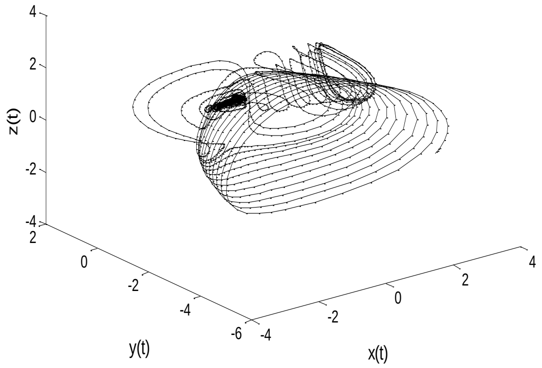
3.2 Newton-Leipnik system

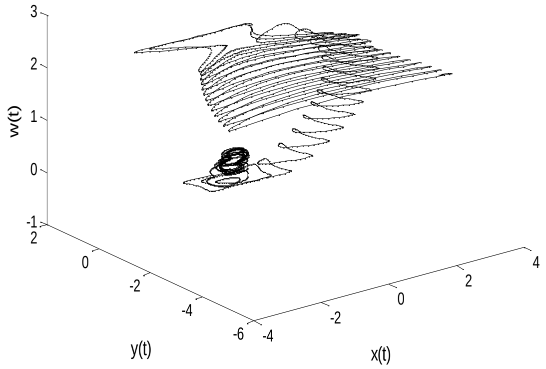
The hyperchaotic Newton-Leipnik system [37] is described by

$$
\begin{aligned}
&D_t x = -a\,x + y + 10\,y\,z + w\\
&D_t y = \; -x - 0.4y + 5\,x\,z\\
&D_t z = b\,z - 5\,x\,y\\
&D_t w = -c\,x\,z + d\,w,
\end{aligned}
\qquad (7)
$$

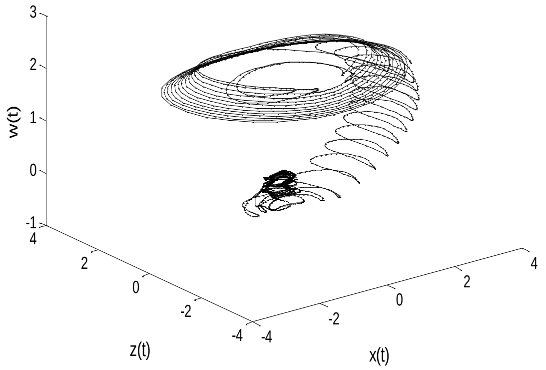
where x, y, z and w are state variables, and a, b, c and d are real constants. At $a = 0.4,\ b = 0.175,\ c = 0.8\ \textit{and}\ d = 0.01,$ system has chaotic behavior. The chaotic attractors of hyperchaotic Newton-Leipnik system are depicted through Fig. 2.

(a) **(b)**





(c) **(d)**



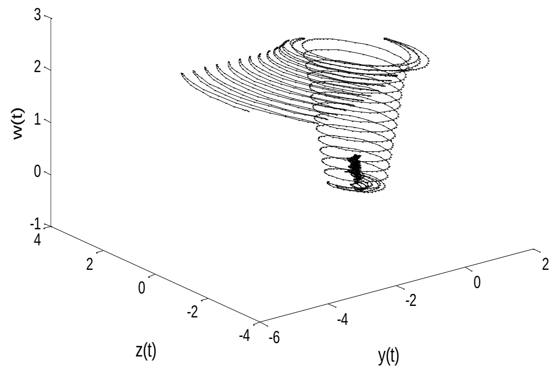


Fig.2. Phase portraits of hyperchaotic Newton-Leipnik attractor in (a) *x*-*y*-*z* space *(b)* *x*-*y*-*w* space (c) x-*z*-*w* space and (d) y-*z*-*w* space.

3.3 Rossler system

The hyperchaotic Rossler system [38] is described by

$$
\begin{aligned}
&D_t x = -y - z \\
&D_t y = x + p\,y + w \\
&D_t z = q + x\,z \\
&D_t w = -r\,z + s\,w\,,
\end{aligned}
\tag{8}
$$

where $x,\ y,\ z, w$ are state variables and $p,\ q, r, s$ are the real parameters. As and when $p = 0.25,\ q = 3,\ r = 0.5,\ s = 0.05$, the dynamic behavior of the system is hyperchaotic. The hyperchaotic attractors in $x - y - z,\ x - y - w,\ x - z - w$ and $y - z - w$ spaces are depicted through Fig. 3.

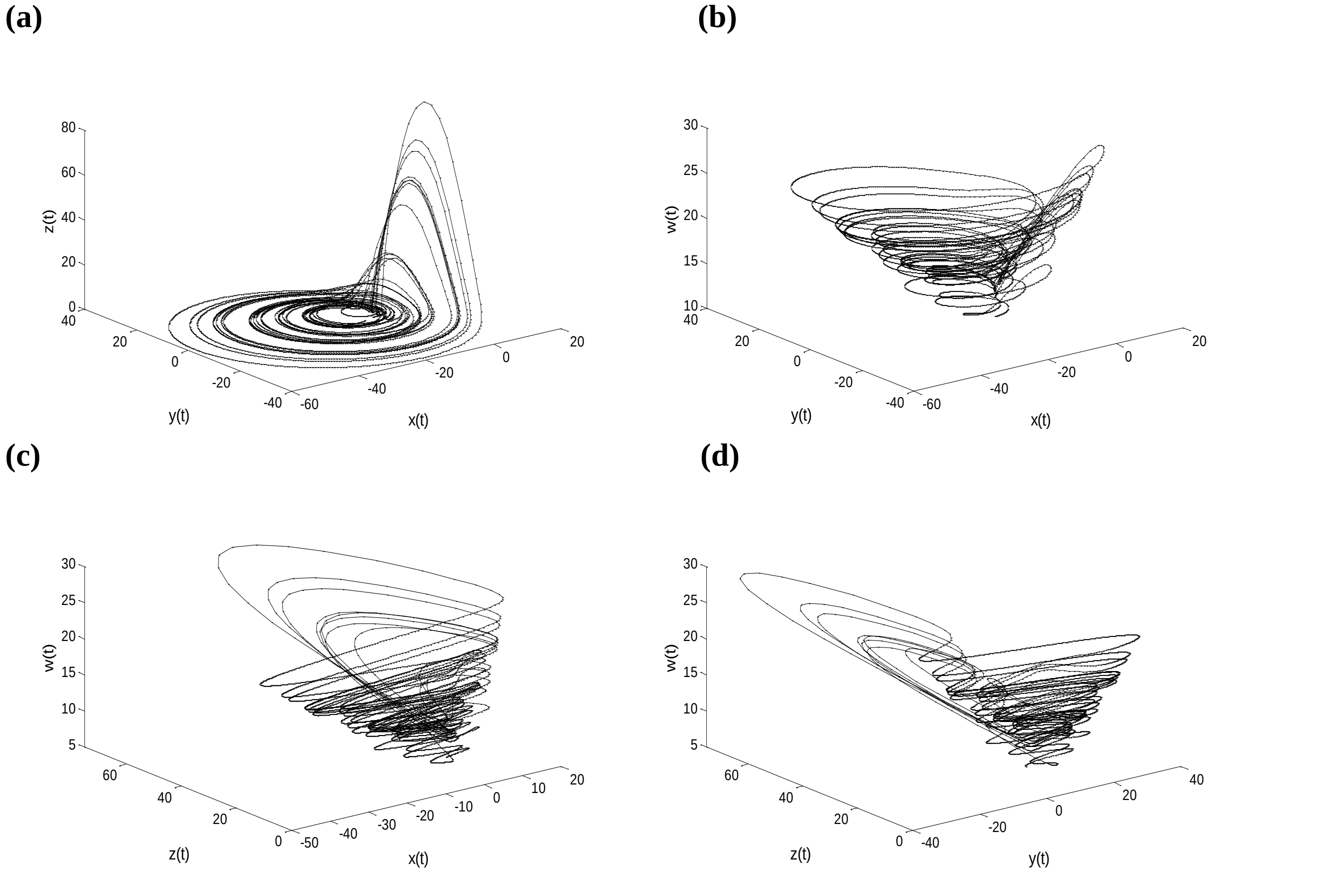


Fig.3. Phase portraits of hyperchaotic Rossler attractor in (a) *x-y-z* space *(b) x-y-w* space (c) x-*z*-*w* space and (d) y-*z*-*w* space.

4. Adaptive anti-synchronization between Lu and Newton-Leipnik hyperchaotic systems

In this section, the anti-synchronization between Lu hyperchaotic system (9) and hyperchaotic Newton-Leipnik system (10) is studied, we assume that Lu system with four unknown parameters drives the Newton-Leipnik system with four unknown parameters.

The drive system is given by

$$
\begin{aligned}
D_t x_1 &= a(y_1 - x_1) + w_1 \\
D_t y_1 &= -x_1 z_1 + c\, y_1 \\
D_t z_1 &= x_1 y_1 - b\, z_1 \\
D_t w_1 &= x_1 z_1 + d\, w_1,
\end{aligned}
\tag{9}
$$

The response system is described by

$$
\begin{aligned}
D_t x_1 &= -p\, x_2 + y_2 + 10\, y_2 z_2 + w_2 + \mu_1(t) \\
D_t y_1 &= -x_2 - 0.4 y_2 + 5 x_2 z_2 + \mu_2(t) \\
D_t z_1 &= q\, z_2 - 5 x_2 y_2 + \mu_3(t) \\
D_t w_1 &= -r\, x_2 z_2 + s\, w_2 + \mu_4(t),
\end{aligned}
\tag{10}
$$

where $\mu(t) = [\mu_1(t),\ \mu_2(t),\ \mu_3(t), \mu_4(t)]^T$ are four control functions to be designed. In order to determine the control functions to realize the anti-synchronization between systems (9) and (10), we add equation (10) to (9) and obtain

$$\begin{aligned} D_t e_1 &= a(y_1 - x_1) + w_1 - p\,x_2 + y_2 + 10\,y_2\,z_2 + w_2 + \mu_1(t) \\ D_t e_2 &= -x_1\,z_1 + c\,y_1 - x_2 - 0.4 y_2 + 5\,x_2\,z_2 + \mu_2(t) \\ D_t e_3 &= x_1\,y_1 - b\,z_1 + q\,z_2 - 5\,x_2\,y_2 + \mu_3(t) \\ D_t e_4 &= x_1\,z_1 + d\,w_1 - r\,x_2\,z_2 + s\,w_2 + \mu_4(t), \end{aligned} \tag{11}$$

where $e_1 = x_2 + x_1,\ e_2 = y_2 + y_1,\ e_3 = z_2 + z_1,\ e_4 = w_2 + w_1.$ Our main aim is to find proper control functions $\mu_i(t), (i = 1,2,3,4)$ and parameter update rule, such that system (10) globally anti-synchronizes the system (9) asymptotically , i.e., $\lim\limits_{t\to\infty} \|e(t)\| = 0.$ For two systems (9) and (10) without controls $(\mu_i(t) = 0, i = 1,2,3,4)$, if the initial condition $(x_1(0), y_1(0), z_1(0), w_1(0)) \neq (x_2(0), y_2(0), z_2(0), w_2(0))$, then the trajectories of two systems will quickly separate each other and become irrelevant. However, when controls are applied, the two systems will approach for anti-synchronization for any initial conditions by appropriate control functions. With this idea, we propose the following adaptive control law for system equation (10) as

$$\begin{aligned} \mu_1(t) &= -\hat{a}(y_1 - x_1) - w_1 + \hat{p}\,x_2 - y_2 - 10\,y_2\,z_2 - w_2 - k_1\,e_1 \\ \mu_2(t) &= x_1\,z_1 - \hat{c}\,y_1 + x_2 + 0.4 y_2 - 5\,x_2\,z_2 - k_2 e_2 \\ \mu_3(t) &= -x_1\,y_1 + \hat{b}\,z_1 - \hat{q}\,z_2 + 5\,x_2\,y_2 - k_3 e_3 \\ \mu_4(t) &= -x_1\,z_1 - \hat{d}\,w_1 + \hat{r}\,x_2\,z_2 - \hat{s}\,w_2 - k_3 e_4 \end{aligned} \tag{12}$$

and parameters update rule for seven unknown parameters $a, b, c, d,\ p,\ q, r, s$ are

$$\begin{cases} \dot{\hat{a}} = (y_1 - x_1)e_1,\ \dot{\hat{b}} = -z_1\,e_3,\ \dot{\hat{c}} = y_1\,e_2,\ \dot{\hat{d}} = w_1\,e_4, \\ \dot{\hat{p}} = -x_2\,e_1,\ \dot{\hat{q}} = z_2\,e_3,\ \ \dot{\hat{r}} = -x_2\,z_2\,e_4,\ \dot{\hat{s}} = w_2\,e_4, \end{cases} \tag{13}$$

where $k_i\,(i = 1, 2, 3, 4)$ are positive real scalars and $\hat{a}, \hat{b}, \hat{c}, \hat{d},\ \hat{p}, \hat{q}\ \hat{r},\ \hat{s}$ are estimates values of $a, b, c, d,\ p,\ q, r, s$ respectively.

Theorem 1. For any initial conditions, the two systems (9) and (10) are globally asymptotically anti-synchronized by adaptive control law (12) and parameter update rule (13).

Proof. Equation (12) with equation (11) yields the error dynamics as

$$\begin{aligned} D_t e_1 &= \bar{a}(y_1 - x_1) - \bar{p}\,x_2 - k_1\,e_1 \\ D_t e_2 &= \bar{c}\,y_1 - k_2\,e_2 \\ D_t e_3 &= -\bar{b}\,z_1 + \bar{q}\,z_{21} - k_3\,e_3 \\ D_t e_4 &= \bar{d}\,w_1 - \bar{r}\,x_2\,z_2 + \bar{s}\,w_2 - k_4\,e_4, \end{aligned} \tag{14}$$

where $\bar{a} = a - \hat{a},\ \bar{b} = b - \hat{b},\ \bar{c} = c - \hat{c},\ \bar{d} = d - \hat{d},\ \bar{p} = p - \hat{p},\ \bar{q} = q - \hat{q},\ \bar{r} = r - \hat{r},\ \bar{s} = s - \hat{s}.$

Consider the following Lyapunov function

$$V(t)=\frac{1}{2}(e^{T}e+\bar{a}^{2}+\bar{b}^{2}+\bar{c}^{2}+\bar{d}^{2}+\bar{p}^{2}+\bar{q}^{2}+\bar{r}^{2}+\bar{s}^{2}). \tag{15}$$

The time derivative of V along the solution of error dynamical systems gives

$$\begin{aligned}
\dot{V}(t)&=e^{T}\dot{e}+\bar{a}\dot{\bar{a}}+\bar{b}\dot{\bar{b}}+\bar{c}\dot{\bar{c}}+\bar{d}\dot{\bar{d}}+\bar{p}\dot{\bar{p}}+\bar{q}\dot{\bar{q}}+\bar{r}\dot{\bar{r}}+\bar{s}\dot{\bar{s}}\\
&=e_1\dot{e}_1+e_2\dot{e}_2+e_3\dot{e}_3+e_4\dot{e}_4+\bar{a}(-\dot{\hat{a}})+\bar{b}(-\dot{\hat{b}})+\bar{c}(-\dot{\hat{c}})+\bar{d}(-\dot{\hat{d}})+\bar{p}(-\dot{\hat{p}})+\bar{q}(-\dot{\hat{q}})\\
&\qquad+\bar{r}(-\dot{\hat{r}})+\bar{s}(-\dot{\hat{s}})\\
&=e_1[\bar{a}(y_1-x_1)-\bar{p}x_2-k_1e_1]+e_2[\bar{c}y_1-k_2e_2]+e_3[-\bar{b}z_1+\bar{q}z_{21}-k_3e_3]\\
&\quad+e_4[\bar{d}w_1-\bar{r}x_2z_2+\bar{s}w_2-k_4e_4]+\bar{a}(-(y_1-x_1)e_1)+\bar{b}(z_1e_3)+\bar{c}(-y_1e_2)\\
&\quad+\bar{d}(-w_1e_4)+\bar{p}(x_2e_1)+\bar{q}(-z_2e_3)+\bar{r}(x_2z_2e_4)+\bar{s}(-w_2e_4)\\
&=-k_1e_1^2-k_2e_2^2-k_3e_3^2-k_4e_4^2\\
&=-e^{T}Pe\le 0\,,
\end{aligned} \tag{16}$$

where

$$e=\begin{bmatrix}e_1\\e_2\\e_3\\e_4\end{bmatrix},\qquad P=\begin{bmatrix}k_1&0&0&0\\0&k_2&0&0\\0&0&k_3&0\\0&0&0&k_4\end{bmatrix}$$

Since $\dot{V}$ is negative semi definite, then $e_1, e_2, e_3, e_4, \hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{p}, \hat{q}, \hat{r}, \hat{s}\in L_\infty$. From error system (14), we have $\dot{e}_1, \dot{e}_2, \dot{e}_3, e_4 \in L_\infty$. Since $\dot{V}(t)=-e^{T}Pe$ and P is a positive definite matrix, then we have

$$\int_0^t \lambda_{\min}(P)\|e\|^2\,dt\le\int_0^t e^{T}Pe\,dt=\int_0^t -\dot{V}\,dt=V(0)-V(t)\le V(0), \tag{17}$$

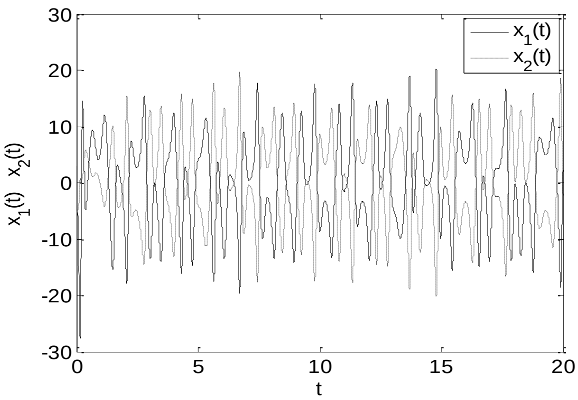
where, $\lambda_{\min}(p)$ is the minimum eigenvalue of positive-definite matrix P. Thus $\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4 \in L_2$. According to the Barbalats lemma, we have $e_1(t), e_2(t), e_3(t), e_4(t)\to 0$ as $t\to\infty$. Therefore, response system (10) can globally anti-synchronize the drive system (9) asymptotically. This completes the proof.

4.1 Numerical simulation and results

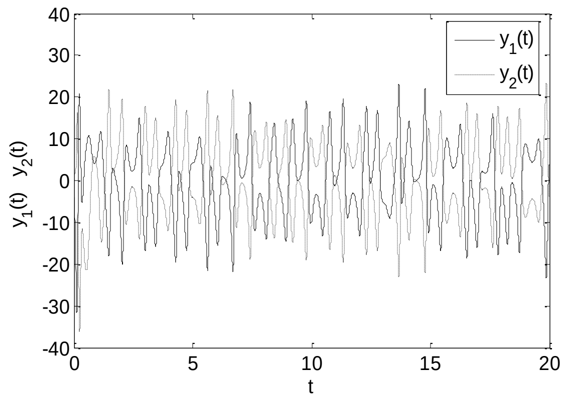
We verify and demonstrate the effectiveness of the proposed method, and discuss the simulation result for the anti-synchronization behavior of two different hyperchaotic Lu and Newton-Leipnik systems. In numerical simulations, the parameters of hyperchaotic Lu and Newton-Leipnik systems are taken as $(a,b,c,d)=(36,3,20,1)$ and $(p,q,r,s)=(0.4,\ 0.175,\ 0.8, 0.01)$ respectively, such that both the systems exhibit hyperchaotic behavior. The initial values of the drive and response systems are taken as $((x_1(0), y_1(0), z_1(0), w_1(0))=(3,-4,-1,2)$ and $((x_2(0), y_2(0), z_2(0), w_2(0))=(-4,2,2,-3))$

respectively. Thus, the initial errors will be $(e_1(0), e_2(0), e_3(0), e_4(0)) = (-1, -2, 1, -1)$. The fourth order Range-Kutta method is used to solve the two systems of equations (9) and (10) with time step size is taken as 0.001. We assume that control inputs $(k_1, k_2, k_3) = (1,1,1)$. Anti-synchronization of systems (9) and (10) via adaptive control laws (12) and parameter update rule (13) with the initial estimated parameters $(\hat{a}(0), \hat{b}(0), \hat{c}(0), \hat{d}(0)) = (-5, -1, -4, 6)$ and $(\hat{p}(0), \hat{q}(0), \hat{r}(0), \hat{s}(0)) = (2, 1, -4, -2)$ are shown in Fig.4 and Fig.5. Fig.4 shows the state response and also the anti-synchronization error system (14) converges to zero. Fig.5 shows that the estimated values $(\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t))$ and $(\hat{p}(t), \hat{q}(t), \hat{r}(t), \hat{s}(t))$ of unknown parameters of the systems (9) and (10) converge to $(a, b, c, d) = (36, 3, 20, 1)$ and $(p, q, r, s) = (0.4,\ 0.175,\ 0.8, 0.01)$ respectively as $t \to \infty$.

(a)



(b)



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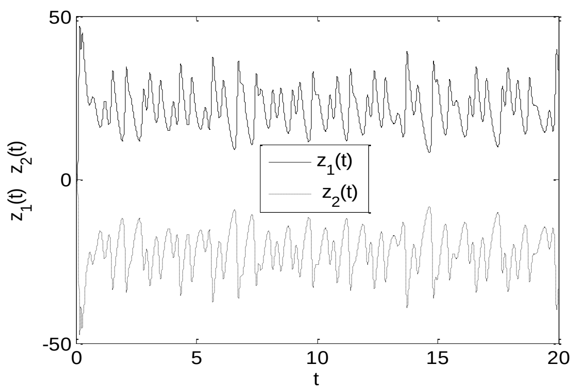

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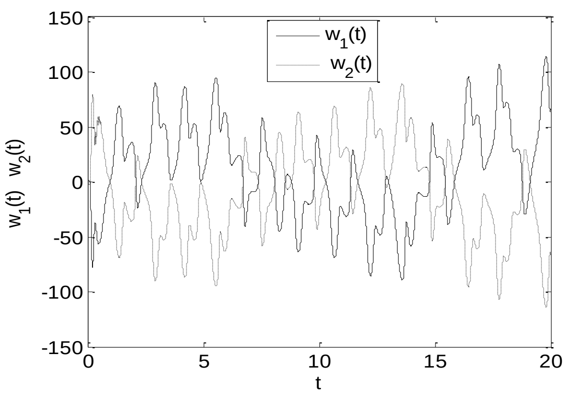


(e)

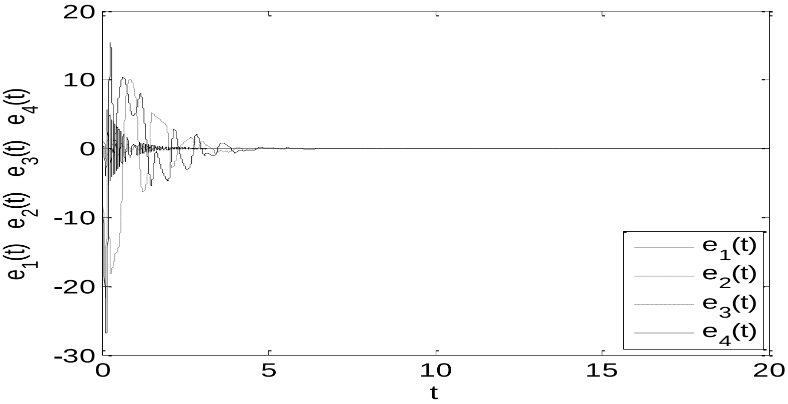


Fig.4. Adaptive anti-synchronization of drive system (9) and response system (10): (a) between $x_1 - x_2$ signals (b) between $y_1 - y_2$ signals (c) between $z_1 - z_2$ signals (d) between $w_1 - w_2$ signals and (e) The error functions $e_1(t), e_2(t)$, $e_3(t)$ and $e_4(t)$ of the hyperchaotic Lu and Newton-Leipnik systems under the controller (12) and the parameters update law (13) with time *t*.

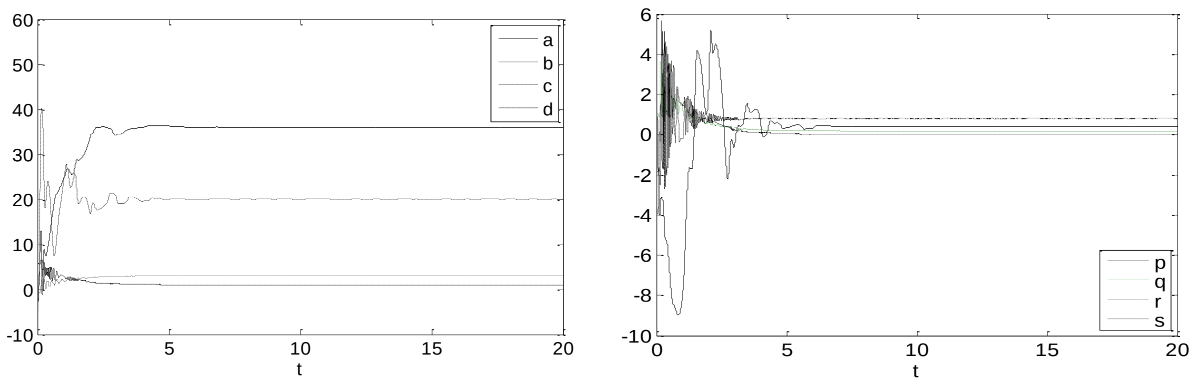


Fig.5. Estimate values of parameters a, b, c, d and p, q, r, s of hyperchaotic Lu and Newton-Leipnik systems with parameter update rule (13).

5. Adaptive anti-synchronization between Newton-Leipnik and Rossler hyperchaotic systems

In this section, the anti-synchronization between Newton-Leipnik hyperchaotic system (18) and hyperchaotic Rossler system (19) is studied, we assume that Newton-Leipnik system with four unknown parameters drives the Rossler system with four unknown parameters.

The drive system is given by

$$\begin{aligned} D_t x_1 &= -a\, x_1 + y_1 + 10\, y_1\, z_1 + w_1 \\ D_t y_1 &= -x_1 - 0.4 y_1 + 5\, x_1\, z_1 \\ D_t z_1 &= b\, z_1 - 5\, x_1\, y_1 \\ D_t w_1 &= -c\, x_1\, z_1 + d\, w_1, \end{aligned} \tag{18}$$

The response system is described by

$$\begin{aligned} D_t x_2 &= -y_2 - z_2 + \mu_1(t) \\ D_t y_2 &= x_2 + p\, y_2 + w_2 + \mu_2(t) \\ D_t z_2 &= q + x_2 z_2 + \mu_3(t) \\ D_t w_2 &= -r\, z_2 + s\, w_2 + \mu_4(t), \end{aligned} \tag{19}$$

where $\mu(t)=[\mu_1(t),\ \mu_2(t),\ \mu_3(t),\mu_4(t)]^T$ are four control functions to be designed. In order to determine the control functions to realize the anti-synchronization between systems (18) and (19), we add equation (19) to (18),

$$
\begin{aligned}
&D_t e_1 = -a\,x_1 + y_1 + 10\,y_1\,z_1 + w_1 - y_2 - z_2 + \mu_1(t)\\
&D_t e_2 = -x_1 - 0.4 y_1 + 5\,x_1\,z_1 + x_2 + p\,y_2 + w_2 + \mu_2(t)\\
&D_t e_3 = b\,z_1 - 5\,x_1\,y_1 + q + x_2 z_2 + \mu_3 t)\\
&D_t e_4 = -c\,x_1\,z_1 + d\,w_1 - r\,z_2 + s\,w_2 + \mu_4(t),
\end{aligned}
\tag{20}
$$

where $e_1 = x_2 + x_1,\ e_2 = y_2 + y_1,\ e_3 = z_2 + z_1,\ e_4 = w_2 + w_1.$ Our main aim is to find proper control functions $\mu_i(t),\ (i=1,2,3,4)$ and parameter update rule, such that system (19) globally anti-synchronizes system (18) asymptotically , i.e. $\lim\limits_{t\to\infty}\|e(t)\| = 0.$ For two systems (18) and (19) without controls $(\mu_i(t)=0, i=1,2,3,4),$ if the initial condition $(x_1(0), y_1(0), z_1(0), w_1(0)),\neq$ $(x_2(0), y_2(0), z_2(0), w_2(0)),$ then the trajectories of two systems will quickly separate each other and become irrelevant. However, when controls are applied, the two systems will approach anti-synchronization for any initial conditions by appropriate control functions. With this idea, we propose the following adaptive control law for system equation (19)

$$
\begin{aligned}
&\mu_1(t) = \hat{a}\,x_1 - y_1 - 10\,y_1\,z_1 - w_1 + y_2 + z_2 - k_1 e_1\\
&\mu_2(t) = x_1 + 0.4 y_1 - 5\,x_1\,z_1 - x_2 - \hat{p}\,y_2 - w_2 - k_2 e_2\\
&\mu_3(t) = -\hat{b}\,z_1 + 5\,x_1\,y_1 - \hat{q} - x_2 z_2 - k_3 e_3\\
&\mu_4(t) = \hat{c}\,x_1\,z_1 - \hat{d}\,w_1 + \hat{r}\,z_2 - \hat{s}\,w_2 - k_3 e_4
\end{aligned}
\tag{21}
$$

and parameters update rule for seven unknown parameters $a, b, c, d,\ p,\ q, r, s$

$$
\begin{cases}
\dot{\hat{a}} = -x_1 e_1,\ \dot{\hat{b}} = z_1\,e_3,\ \dot{\hat{c}} = -x_1\,z_1 e_4,\ \dot{\hat{d}} = w_1\,e_4\\
\dot{\hat{p}} = y_2\,e_2,\ \dot{\hat{q}} = e_3,\ \dot{\hat{r}} = -z_2\,e_4,\ \dot{\hat{s}} = w_2\,e_4
\end{cases}
\tag{22}
$$

where $k_i\ (i=1,2,3,4)$ are positive real scalars and $\hat{a}, \hat{b}, \hat{c}, \hat{d},\ \hat{p}, \hat{q}\ \hat{r},\ \hat{s}$ are estimates values of $a, b, c,$ $d,\ p,\ q, r\ s$ respectively.

Theorem 2. For any initial conditions, the two systems (18) and (19) are globally asymptotically anti-synchronized by adaptive control law (21) and parameter update rule (22).

Proof. Equation (21) with equation (20) yields the error dynamics as

$$
\begin{aligned}
&D_t e_1 = -\bar{a}\,x_1 - k_1\,e_1\\
&D_t e_2 = \bar{p}\,y_2 - k_2\,e_2\\
&D_t e_3 = b\,z_1 + \bar{q} - k_3\,e_3\\
&D_t e_4 = -\bar{c}\,x_1\,z_1 + \bar{d}\,w_1 - \bar{r}\,z_2 + \bar{s}\,w_2 - k_4\,e_4,
\end{aligned}
\tag{23}
$$

where $\bar{a} = a - \hat{a},\ \bar{b} = b - \hat{b},\ \bar{c} = c - \hat{c},\ \bar{d} = d - \hat{d},\ \bar{p} = p - \hat{p},\ \bar{q} = q - \hat{q},\ \bar{r} = r - \hat{r},\ \bar{s} = s - \hat{s}$

Consider the following Lyapunov function

$$V(t)=\frac{1}{2}(e^T e+\bar{a}^2+\bar{b}^2+\bar{c}^2+\bar{d}^2+\bar{p}^2+\bar{q}^2+\bar{r}^2+\bar{s}^2). \tag{24}$$

The time derivative of V along the solution of error dynamical systems gives that

$$\begin{aligned}
\dot{V}(t)\dot{V}(t) &= e^T\dot{e}+\bar{a}\,\dot{\bar{a}}+\bar{b}\,\dot{\bar{b}}+\bar{c}\,\dot{\bar{c}}+\bar{d}\,\dot{\bar{d}}+\bar{p}\,\dot{\bar{p}}+\bar{q}\,\dot{\bar{q}}+\bar{r}\,\dot{\bar{r}}+\bar{s}\,\dot{\bar{s}} \\
&= e_1\dot{e}_1+e_2\dot{e}_2+e_3\dot{e}_3+e_4\dot{e}_4+\bar{a}\,(-\dot{\hat{a}})+\bar{b}\,(-\dot{\hat{b}})+\bar{c}\,(-\dot{\hat{c}})+\bar{d}\,(-\dot{\hat{d}})+\bar{p}\,(-\dot{\hat{p}}) \\
&\quad+\bar{q}\,(-\dot{\hat{q}})+\bar{r}\,(-\dot{\hat{r}})+\bar{s}\,(-\dot{\hat{s}}) \\
&= e_1[-\bar{a}\,x_1-k_1\,e_1]+e_2[\,\bar{p}\,y_2-k_2\,e_2]+e_3[b\,z_1+\bar{q}-k_3\,e_3]+e_4[-\bar{c}\,x_1\,z_1 \\
&\quad+\bar{d}\,w_1-\bar{r}\,z_2+\bar{s}\,w_2-k_4\,e_4]+\bar{a}\,(x_1e_1)+\bar{b}\,(-z_1\,e_3)+\bar{c}\,(x_1\,z_1e_4)+\bar{d}\,(-w_1\,e_4) \\
&\quad+\bar{p}(-y_2\,e_2)+\bar{q}\,(-e_3)+\bar{r}\,(z_2\,e_4)+\bar{s}\,(-w_2\,e_4) \\
&= -k_1e_1^2-k_2e_2^2-k_3e_3^2-k_4e_4^2 \\
&= -e^T P\,e\le 0\,,
\end{aligned} \tag{25}$$

where

$$e=\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}, \qquad P=\begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}$$

Since $\dot{V}$ is negative semi definite. Then $e_1,\, e_2,\, e_3, e_4, \hat{a},\, \hat{b},\, \hat{c}, \hat{d},\, \hat{p},\, \hat{q},\, \hat{r}, \hat{s}\in L_\infty$. From error system (23), we have $\dot{e}_1,\, \dot{e}_2,\, \dot{e}_3, \dot{e}_4\in L_\infty$ Since $\dot{V}(t)=-e^T P\,e$ and P is a positive definite matrix, then we have

$$\int_0^t \lambda_{\min}(P)\|e\|^2\,dt\le\int_0^t e^T P\,e\,dt=\int_0^t -\dot{V}\,dt=V(0)-V(t)\le V(0), \tag{26}$$

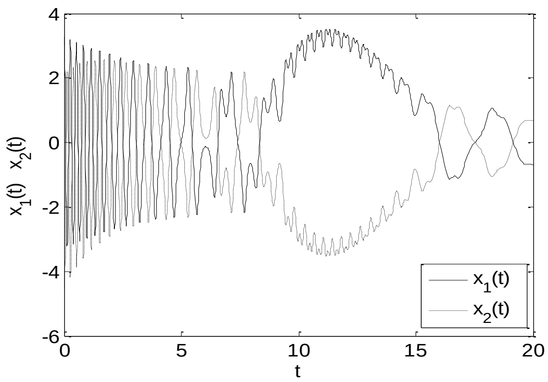
where, $\lambda_{\min}(p)$ is the minimum eigenvalue of positive-definite matrix P. Thus $\dot{e}_1,\, \dot{e}_2,\, \dot{e}_3, \dot{e}_4\in L_2$. According to the Barbalats lemma, we have $e_1(t),\, e_2(t),\, e_3(t), e_4(t)\to 0$ as $t\to\infty$.Therefore, response system (19) can globally anti-synchronized the drive system (18) asymptotically. This completes the proof.

5.1 Numerical simulation and results

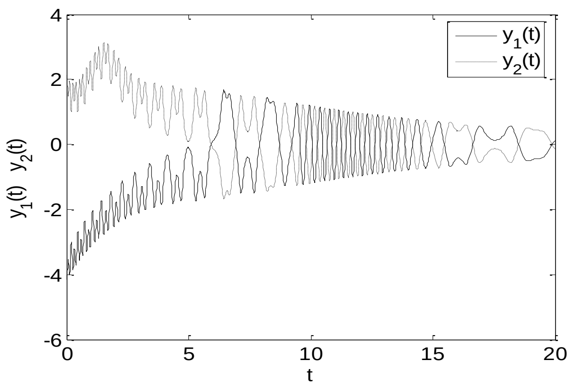
We verify and demonstrate the effectiveness of the proposed method, and discuss the simulation result for the anti-synchronization behavior of two different hyperchaotic Newton-Leipnik and Rossler systems. In numerical simulations, the parameters of hyperchaotic Newton-Leipnik and Rossler systems are taken as $(a,b,c,d)=(0.4,\ 0.175,\ 0.8, 0.01)$ and $(p,q,r,s)=(0.25,\ 3,\ 0.5, 0.05)$ respectively, such that both the systems exhibit hyperchaotic behavior. The initial values of the drive and response systems are taken as $((x_1(0), y_1(0), z_1(0), w_1(0))=(3,-4,-1,2)$ and $(x_2(0), y_2(0), z_2(0), w_2(0))=(-4,2,2,-3)$ respectively. Thus, the initial errors will be

$(e_1(0), e_2(0), e_3(0), e_4(0)) = (-1, -2, 1, -1)$. The fourth order Range-Kutta method is used to solve the two systems of equations (18) and (19) with time step size is taken as 0.001. We assume that control inputs $(k_1, k_2, k_3) = (1,1,1)$. Anti-synchronization of systems (18) and (19) via adaptive control laws (21) and parameter update rule (22) with the initial estimated parameters $(\hat{a}(0), \hat{b}(0), \hat{c}(0), \hat{d}(0)) =$ $(-5, -1, -4, 6)$ and $(\hat{p}(0), \hat{q}(0), \hat{r}(0), \hat{s}(0)) = (2, 1, -4, -2)$ are shown in Fig.6 and Fig.7. Fig.6 shows the state response and also the anti-synchronization error system (23) converges to zero. Fig.7 shows that the estimated values $(\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t))$ and $(\hat{p}(t), \hat{q}(t), \hat{r}(t), \hat{s}(t))$ of unknown parameters of the systems (18) and (19) converge to $(a, b, c, d) = (0.4,\ 0.175,\ 0.8, 0.01)$ and $(p, q, r, s) = (0.25,\ 3,\ 0.5, 0.05)$ respectively as $t \to \infty$.

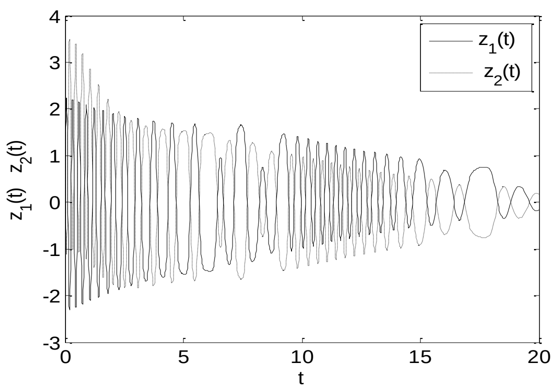
(a)



(b)



(c)



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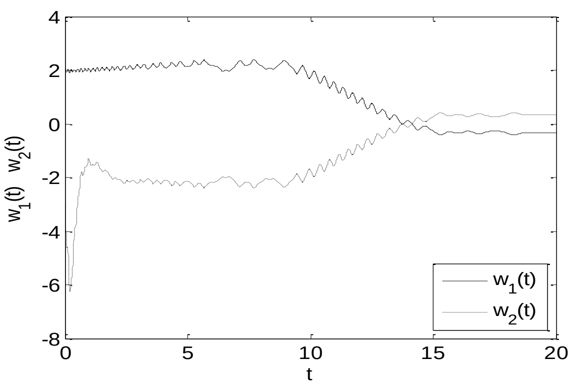

(e)

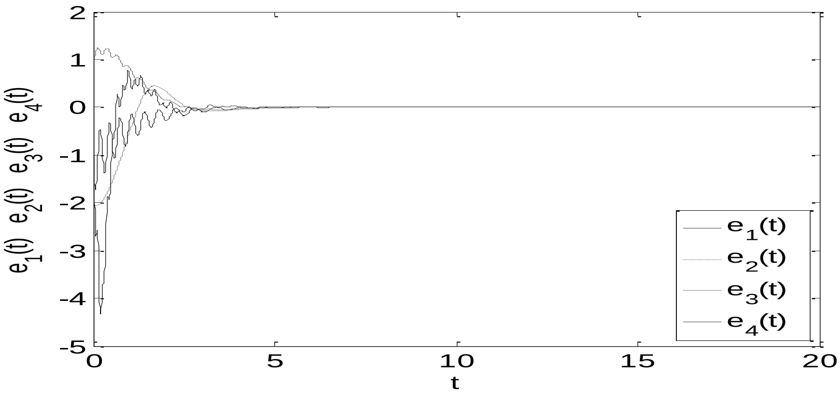

Fig.6. Adaptive anti-synchronization of drive system (18) and response system (19): (a) between $x_1 - x_2$ signals (b) between $y_1 - y_2$ signals (c) between $z_1 - z_2$ signals (d) between $w_1 - w_2$ signals and (e) The error functions $e_1(t), e_2(t), e_3(t)$ and $e_4(t)$ of the hyperchaotic Newton-Leipnik and Rossler systems under the controller (21) and the parameters update law (22) with time *t*.

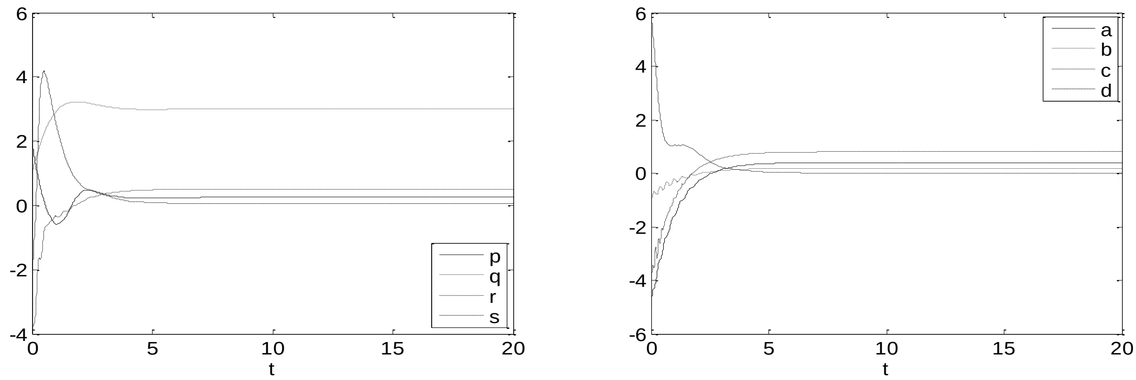

Fig.7. Estimate values of parameters a, b, c, d and p, q, r, s of hyperchaotic Newton-Leipnik and Rossler systems with parameter update rule (22).

6. Conclusion

The present investigation has attained accomplishment in two significant capacities. First it is successfully carried out the study of anti-synchronization between Lu and Newton-Leipnik hyperchaotic systems, and Newton-Leipnik and Rossler hyperchaotic systems with uncertain parameters using adaptive control method. Adaptive controller and parameters update law are designed properly to anti-synchronize two different pair of hyperchaotic systems based on the Lyapunov stability theorem. The second one is the numerical simulation, which are carried out using Runge-Kutta method calls for appreciation to show that the method is reliable and effective for adaptive anti-synchronization of nonlinear dynamical systems.

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