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HYBRID SYSTEMS AND RANDOMIZED MEASURING IN NONEQUILIBRIUM PROCESSES

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Abstract

Recently in the control theory the hybrid systems are developed actively for the description of complicated dynamic processes. The word "hybrid" is underlined that the description of the system consist of two levels. One level presents the dynamic behaviour of the system in a state space (high-rate process) and the other one is the structure evolution of the state space itself (slow process). The paper is devoted to the growing role of the systematic error that cannot already be excluded from the consideration. Special feature of nonequilibrium processes is a necessity to take into account the systematic error in the models with the changing state space structure. The conventional methods are

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not adequate to take efforts for the systematic error depression. The main goal of the paper is to demonstrate possibilities to investigate nonequilibrium processes in complicated systems without the standard assumptions respectively the observation noises due to randomization of the observation process (active observations).

1 Mathematical models of processes and measurements

Till present the control theory cannot propose any satisfactory solutions to numerous practical problems. Among them there are problems on the dynamically reconfigurated intellectual control, asynchronic control, the control via Internet, the control of a robot's team, the control of systems with spontaneous switching, programming of a system for a bacteria's control and many others. One of the main difficulties is closely connected to the deficiencies of the conventional approaches to the construction of the formal mathematical models of the physical processes. The conventional methods to describe dynamical systems suppose a space of the system states chosen $\mathcal{X} \subset \mathbb{R}^n$, $(n \in \mathbb{N})$ and a dynamical equation constructed

$$\dot{x} = F(t, \theta, x, u) \qquad (1)$$

governing a time depending variable of the state $x \in \mathcal{X}$. The control variable $u(t) \in U \subset \mathbb{R}^m$ characterizes an external influence on the system and plays a role of a medium between the system and its surroundings. The form of the dynamical equations and boundary conditions is usually given within a finite size set of constants θ called parameters of the system. After the satisfactory and substantiated choice of the mathematical model one of the most important problems appearing in the analysis of the real dynamic systems, is a problem on getting and a refinement of an information respectively the real values of the system parameters (to identify the system). This information is usually used to predict the system behaviour and to construct the control law. Together with the mathematical model of a dynamical process in control systems an observation model is chosen. Any measurements always give mean values of variables averaged over both a space region and a time interval. Usually the observation set can be presented as follows

$$y(t) = \int_{t-\delta}^{t} dt' \int_{M} D(t', x, u) dx, \qquad (2)$$

where is a subset of the state space (usually $M \subset \partial\mathcal{X}$) a tine interval δ are determined by the characteristics of the registering instruments. After the discretisation of the observation model one usually get an expression

$$y_t = D_t(x_t, u_t) + \bar{v}_t + \hat{v}_t(\theta), \tag{3}$$

where $v_t = \bar{v}_t + \hat{v}_t(\theta)$ is understood as a standard observation error (noise), the first unit $\bar{v}_t$ is usually called the statistical error and the second one $\hat{v}_t(\theta)$ is the systematic error (the model error) or jamming. For the last 50 years there were very many papers dedicated to the system identification problems. In physics the problem on the reconstruction of the parameter θ basing on the observation data is called the inverse problem. In scope of the most part of the conventional methods the statistical error is considered either being small or having any useful properties, for example, being bounded. As to the systematic error, it is not usually considered at all supposing that it can be removed either during the choice of the mathematical model or by the appropriate planning of an experiment (see, for example, [1]). In nonlinear models the solution to the identification problem is often complicated by the circumstance that the system trajectories being initially close to each other, for a finite time interval (sometimes rather small) diverge far away. This effect is the ground for the chaotic description of the dynamic system behaviour (see, for example, the review [2]). The models with chaos present a wide field for applications of the modern mathematical theories. However, the last ones being very interesting and physically profounded, unfortumately don't allow solutions to the identification problem. In general, the transition to chaos suppose a new choice of the mathematical model based on the more rough description level of the macroscopic scale where the mean values are distributed in space. The distribution densities are governed by the balance equations that are partially differential ones near the thermodynamic equilibrium state of the system. It is well known in classical continuum mechanics that at small typical time interval and under not very intensive external influence these equations are of the hyperbolic type while under long intensive loading the equations become parabolic and describe diffusive transport processes in medium. The high-rate and high-gradient processes combining both the wave and diffusive transport properties as the new theoretical [3–5] and experimental researches confirm, cannot be governed by the differential equations of the classical continuum mechanics. In scope of the nonequilibrium statistical mechanics new integro-differential equations had been derived that embrace a range from the wave regime with the not decaying memory and space correlation up to the hydrodynamic regime where the system entirely forgets its initial state. However, such a description is not complete.and requires a structurization of the uncertainty.

Real systems have an infinite number of the degrees of freedom and the supposition that the phase space of the system is finite dimensional, is adequate only if a finite number of the order parameters (usually not large) are derived among all the degrees of freedom that determine the system behaviour with a satisfactory accuracy on the given conditions. The dynamic “transitive” processes proceeding during the high-rate changing of the external conditions are the most difficult for the formal description. The conventional approaches to describe the nonequilibrium processes are not adequate as the structure of the phase space itself become time depending. The experimental observation of the nonequilibrium processes [4] confirm the origination in the system of new internal structures of a mesoscopic scale (intermediate between micro- and macro-) that in great part define the type changing of the formal model. The examples of such structure formation are: clasterisation in flows of concentrated disperse mixtures, multiscale vortical structures formation in turbulent flows, mesoscopic rotations in plastic flows of solid materials under pulse loading (see, for example, [4-5]), and hierarchies of structures in living systems. At present it is known that synergetic processes of the mesoscopic structure formation in open thermodynamic systems are closely connected to the formation in the system of an informational control feedback. The internal control together with the external one via the imposed on the system loading conditions is followed by the space-time discretisation in the nonequilibrium system. The physical carriers of the information in the system are the new dynamic structure elements. Otherwise, in order to describe the dynamic behaviour of nonequilibrium system in a correct way it is not sufficiently to choose the formal model like (1) with the fixed numbers of the order parameters at the only one chosen scale level.

Supposing that the structure of the system state space can evolve with time a new model class is considered. Or more precisely, let a set of the variable structures of the system state space Σ is given. Denote the current structure of the state space $s \in \Sigma$ and $\mathcal{X}(s)$ is the state space. Then instead of Eqn. (1) consider

$$\dot{x} = F_s(t, \theta_s, x, u), \quad x(t) \in \mathcal{X}(s(t)) \subset L, \tag{4}$$

where L is a space possible having an infinite dimension where all intermediate state spaces are included. As a rule, The structure of the state space changes more slowly than the system dynamics itself and the description in the space $\mathcal{X}(s)$ is essentially more simple than the one in the space L. The choice of the spaces $\mathcal{X}(s)$ with the finite number (often rather low) of dimensions is typical.

The physical considerations prescribe an integral dependence of the state space structure on the prehistory of the system evolution

$$s(t) = \int_{-\infty}^{t} dt' \int_{\mathcal{X}(s(t'))} S(s,x)dx$$

In the control theory such type models to describe dynamic processes in the complicated systems had arisen not long ago and called "hybrid" [6]. In this name it is underlined that the description of the system is two-level. One level presents the dynamic behaviour of the system in a state space (high-rate process) and the other one is the structure evolution of the state space itself (slow process). So, the transition to the more general formula (4) is inevitably followed by the growing role of the systematic error $\hat{v}_t(\theta_s)$ that cannot already be excluded from Eqn.(3). The conventional methods are not adequate to take efforts for the systematic error depression as far as according to the new model construction the structure of the state space is not surely known. So, a special feature of nonequilibrium processes is a necessity to take into account the systematic error in the models with the changing state space structure.

2 Randomized measuring

Then some important questions appear: what to do with the systematic error in the inverse problems in physics? Is it generally possible to solve the identification problem taking into account the inevitable existence of the systematic error in the observation data? In order to answer the questions it is necessary to revised the conventional concept of measurement. The main goal of the paper is to demonstrate possibilities to investigate nonequilibrium processes in complicated systems without the standard assumptions respectively the observation noises due to randomization of the observation process (active observations). Paper [7] is just devoted to the estimation problem of unknown parameters at presence of arbitrary noises.

Consider a simple example of habituated algorithm for the measurements data processing. Let one is interesting in a characteristic of the investigated process θ which value is to be controlled. Suppose that it can be determined on the base of some transfer function $G(\theta)$ from the control u to the measurements y. Consider for a simplification a scalar case with $G(\theta) = \theta$. Then the observation model (3) results

$$y = \theta u + \textit{observation noise}.$$

If the system is considered being steady-state in order to define the unknown value of the parameter θ the standard method is used when series of experiments are repeated and the measurements data are averaged as usual. However, this method is applicable only supposing the independence and centering of the observation noise series. When it is impossible to repeat measurements many times, for example, in high-rate dynamic processes the value y resulted in experiment under high level observation noise practically gives no information respectively the real value of the parameter θ. Otherwise, a simple averaging of the observation data is not valid at presence of the systematic noise of the observation model. It may seem strange, but one of the effective way to remove the systematic noise effect is using of the proposed in [7-10] randomized algorithms for active measurements. Suppose that in time moments $n = 1, 2, \ldots$ some probe control actions produced on the system are known $u_1, u_3, \ldots$ denote their set by $\{u_n\}$. Then for the observations $\{y_n\}$ dimensionless relationships follow

$$y_n = \theta u_n + w_n, \ n = 1, 2, \ldots,$$

where $w_1, w_2, \ldots$ are the observation noises. Taking in mind a statistical nature of the control actions $\{u_n\}$ suppose they are a sequence of independent bounded random values with known nonzero mean value $M_u \neq 0$, positive bounded dispersion $\sigma_u^2 > 0$ and bounded forth moment. The idea of the least mean squared method dated still from Gauss and Legendre, is based on averaging of n successive observation data multiplied by the corresponding values of the control actions. According to the strong large numbers law the sequence of estimates in scope of the usual mean squared method at presence of random independent observation noise with bounded statistical moments,

$$\hat{\theta}_n = \frac{\sum_{i=1}^{n} u_i y_i}{\sum_{i=1}^{n} u_i^2}$$

converges at $n \to \infty$ with the probability 1 to the value $\theta + \frac{M_u M_v}{\sigma_u^2}$, where M_u is a mean value of the control actions. Consequently, at rather large number of observations and the known value M_v the problem on the determination of the value θ is solved.

If the value M_v is unknown or the sequence of the observation noises is unknown and probably not random the classical algorithm is not valid. Multiplying by $\Delta_n = (u_n - M_u)$ both parts of the relationship determining the observations, after some simple manipulation one can get

$$\Delta_n y_n = \theta \Delta_n^2 + \theta \Delta_n M_u + \Delta_n w_n.$$

Summarizing and averaging the first N observations results

$$\frac{1}{N}\Delta_n y_n = \theta\frac{1}{N}\Delta_n^2 + \frac{1}{N}(\theta\Delta_n M_u + \Delta_n w_n).$$

The first and the second terms in the right-hand part under the adopted suppositions according to the strong large number law, at $n \to \infty$ with probability 1 tend to $\theta\sigma_u^2$ and zero correspondingly. Just the same can be shown respectively the last unit at $n \to \infty$ that tends to zero. From there it follows that at $u_1 \neq M_u$ the sequence of estimations $\{\hat{\theta}_n\}$, formed by the rule

$$\hat{\theta}_n = \frac{\sum_{i=1}^{n}(u_i - M_u)y_i}{\sum_{i=1}^{n}(u_i - M_u)^2},$$

converges at $n \to \infty$ with probability 1 to θ.

Fig.1 shows the typical behavior of estimates sequences $\{\hat{\theta}_n\}$ on different types of observation noise.

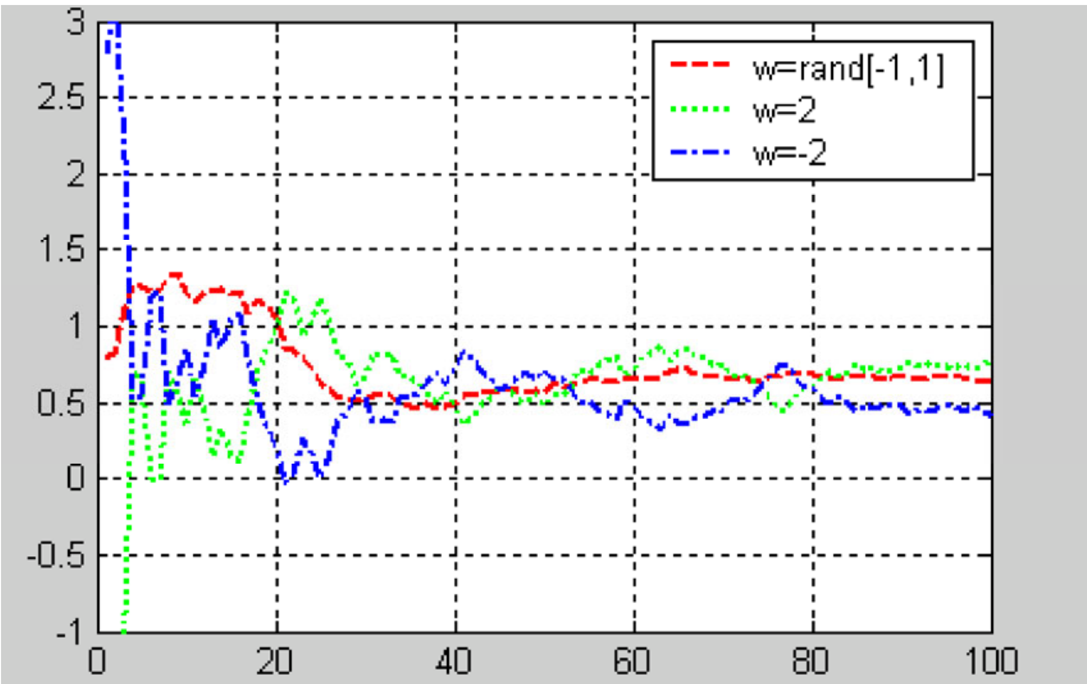

Fig. 1. Sequences of estimates $\{\hat{\theta}_n\}$

The essential deficiency of the method is that the estimates are consistent asymptotically. In between it had been noted above that in nonequilibrium processes the structure of the state space can evolve in time and there is no possibility to observe the system for a long time. Besides, the using of the control actions itself can disturb the system and change its description. In paper [11] an original way to estimate the transfer function from u to y basing on a finite number of observations is proposed. Though this method is derived on the standard suppositions respectively the observation noises it can be extended to the case of arbitrary noises. This allows a hope to develop in future a method to estimate the unknown parameters that valid at arbitrary noises and finite number of observations.

So, the traditional supposition that the noises are independent and centered (have zero-mean values) during observation in series of experiments can be essentially released. It means that it is theoretically possible to compensate the negative effect of the systematic noises by the observation process randomization in control of nonequilibrium processes under high-rate loading conditions. It is worth noting that at present even high-rate transitive processes can be controlled. For example, the shock induced wave front propagating in metal passes a fixed point for a time interval about 10^{-7}s. The modern instruments and microprocessor techniques had reached the level of nanotechnologies (10^{-9}), and entirely allow a possibility to control any process characteristics in real time, Particularly, this can derive from the theoretically predicted "rough" set of probable trajectories (or the model types) the unique one that had realized in the given experiment or at any rate essentially to reduce the uncertainty set. However, it must be underlined that one cannot expect the good replication of the experimental results due to the nonequilibrium character of the probability distributions determining the macroscopic averaged characteristics. Further, the obtained information can be used in the control channel.

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