DIFFERENTIAL EQUATIONS AND CONTROL PROCESSES
N 2, 1998
Electronic Journal,
reg. N P23275 at 07.03.97

http://www.neva.ru/journal
e-mail: diff@osipenko.stu.neva.ru

Group analysis of differential equations

D I G R A N - COMPUTERIZED REFERENCE BOOK NEW GENERATION FOR EXACT SOLUTIONS OF DIFFERENTIAL EQUATIONS

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Abstract.

Computer Look-Up System *DIGRAN* the toolbox for students, post-graduates, teachers and also engineers, recherche, mathematician, which use the mathematical models of complex dynamics process. The system use for solution of the problems of mathematical modelling with this computer algebra systems. All the software work at workstations the firms DEC, CDSystems, Sun Microsystems and IBM PC's. This is a computerized reference book which promotes to specify and find models and its solutions. More than 4000 not-trivial solutions will be present. This software is based on the new original approach to differential equations solving. The basis version contain original solutions nonlinear differential equations first, second and higher kinds, which out with the teoretical-discrete-group methodological.

1 Major DIGRAN benefits and models classes of equations

The collection of systems, united in a DISAM project, is essentially a mathematical consultant on DIscrete Symmetry of Analytical Models, realised for computer and computer algebra systems.

The system DIGRAN (DIscrete-GRoup ANalysis) is based on the new fundamental mathematical theory specially developed in [1] . The number of analytical solutions already obtained with the method from 10 to 100 times exceeds those found in classical reference books, e.g. E.Kamke [1959], G.Murphy [1960], D.Zwillinger [1989]. For example, more than 1500 Abel equations of the second kind and more than 800 nonlinear equations of the 3 order and higher can be solved analytically.

Transformations, symmetries and invariants can be calculated as well.

Base of Models/Solutions is a computerized reference book which promotes to specify and find models and solutions. More than 4000 nottrivial solutions will be present.

Analytical Solver of Models is an interactive computer program which gives user a way to specify a model and then it tries to construct an analytical solution of the model in a dialogue with the user.

Visualization Block is a set of utility programs which helps the user to investigate models and qualitative properties of solutions.

Sample List of Model Classes:

1. Abel equation of the second kind and nonlinear oscillation equations

$$yy' - y = f(x), \tag{1}$$

$$yy' - f(x)y = g(x), \tag{2}$$

$$yy' - f(x)y^2 - g(x)y = h(x) \tag{3}$$

$$y'' + ay' + by = f(y) \tag{4}$$

2. Equations of type (k, r, s)

$$(y')^k = Ay^r + Bx^s \tag{5}$$

$$(y')^k = Ax^{s_1}y^{r_1} + Bx^{s_2}y^{r_2} \tag{6}$$

3. Equations of type (f, g)

$$(y')^k = f(y) + g(x) \tag{7}$$

4. Generalized Emden-Fowler equation

$$y'' = Ax^n y^m (y')^l \tag{8}$$

5. Generalized equations of type (4)

$$y'' = \sum_k A_k x^{n_k} y^{m_k} (y')^{l_k} \tag{9}$$

6. Equations of type (f, g, h)

$$y'' = f(x)g(y)h(y') \tag{10}$$

7. Generalized equations of type (6)

$$y'' = \sum_k f_k(x)g_k(y)h_k(y') \tag{11}$$

8. Generalized boundary layer equation

$$y''' = Ax^\alpha y^\beta (y')^\gamma (y'')^\delta \tag{12}$$

9. Generalized equations of type (8)

$$y''' = \sum_k A_k x^{\alpha_k} y^{\beta_k} (y')^{\gamma_k} (y'')^{\delta_k} \tag{13}$$

This software may be implemented for a C or C++, and system control of DB Clarion.

2 The system structure

The system DIGRAN contain secondary information in system structure, basis content, glossary, references, sphere of application, comparison and software/hardware platforms.

$$(Tabl.symm., 1 - int., diff.alg.) \longleftarrow (Superclass) \longrightarrow (Help)$$

$$(Superclass) \longrightarrow (Class) \longrightarrow (Subclass)$$
$$(Subclass) \longrightarrow (Bushes)$$
$$(Subclass) \longrightarrow (Equations)$$
$$(Bushes) \longrightarrow (Subset.equations) \longrightarrow (Equations) \longrightarrow (Solutions)$$
$$(Subset.equations) \longrightarrow (Solutions)$$

3 Examples.

EXAMPLE 1. The information approach to problems of modeling nonlinear systems with a priori symmetry.

The information equipment of the object domain by modeling nonlinear systems at the expense of special metamathematical description allow, on the one hand, consider the algorithms analysis symmetries and synthesis like the algorithms of the production rule on the expert systems and other on the information domain introduce "lattice concept" for utilization new information knowledge base like the supplementary computer look-up systems. Then the problem of resolution on closed view for nonlinear systems, describing ordinary differential equations, reformatted like the problem invariance, just exactly, the commutative of the diagram of information- object relations [2] . Here, for example, $X \equiv F(x, y, y', y'', \vec{a})$ – the initial system with vector of the parameters $\vec{a} \in D$; $Y \equiv F_1(t, u, \dot{u}, \ddot{u}, \vec{b})$ – the transformation system with vector $\vec{b} \in D$; $Z \equiv (\phi_1(\tau), \psi_1(\tau))$ – the multitude of recherche solutions of the transformation equation belong of differential ring 3 rank, give of parametric performance of the general solution of the initial equation $(\tau, \phi(\tau), \psi(\tau))$ $A_3[\tau, \phi(\tau), \psi(\tau)]$.

$$\hat{X} = X \times L_x^1 \times L_x^2 \times L_x^3; \qquad \hat{Y} = Y \times L_y; \qquad \hat{Z} = Z \times L_z,$$

where L_x^1 – the classification lattice of the types equations; L_x^2 – of the forms transformations; L_y – of the forms solutions (bush); L_z – of the types functions on A_3; L_x^3 – the structure - classification lattice admit on space $R(D)$ of the group (a priori symmetry of the model).

$$r_1 : \hat{X} \longrightarrow X;$$

r_2 and r_3 – analogize. For $c_1 : F \rightarrow F_1$; if c_2 – operator integrating then ultraoperator resolution take the form:

$$\hat{c} = \hat{c}_2 \circ \hat{c}_1.$$

EXAMPLE 2. Equations of first order.

$Superclass$ – Differential equation of first order.

$(Class)_i$ – Abel equations of the second kind $yy' - y = R(x)$.

$(Subclass)_j$ – Equations of type $yy' - y = R_j$ $(j = 1, 2, ...)$:

$$yy' - y = R_1 = sx + Ax^m,$$
$$yy' - y = R_2 = sx + \sigma A(\alpha A^2 x^{-1/2} + \beta A + \gamma x^{1/2}),$$
$$yy' - y = R_3 = sx + \alpha\sqrt{x^2+\gamma} + \beta/\sqrt{x^2+\gamma},$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

where $s, m, \sigma, \alpha, \beta, \gamma$ – the parameters of the groups, A – the arbitrary parameter.

Or, equations on the forme parametric:

$$pp'_\xi - p = \frac{(m+l-1)u^2 - (m+2l-n-3)u - (n-l+2)}{(m+2l-3)u + (n-2l+3)} u^{1-l},$$

$$\xi = (\frac{m+2l-3}{2-l}u + \frac{n-2l+3}{1-l})u^{1-l}.$$

EXAMPLE 3. Equations of second order.

$Superclass$ – Differential equation of second order.

$(Class)_i$ – Generalized Emden-Fowler's equation

$$y'' = Ax^n y^m y'^l.$$

$(Subclass)_j$ – Set of the equations $[n, m, l]$.

$(Bushes)_k$ – Bush of polynomial functions [3, 4, 5] , bush of EULER functions, bush of trigonometric functions, bush of elliptic integrals of the second kind,...

Bush of polynomial functions and subset of the equations corresponding:

$n = 0$, $m = -1/2$, $l = 0$

$$y'' = Ay^{-1/2} \tag{14}$$

$n = 1$, $m = 0$, $l = 0$

$$y'' = Ax \tag{15}$$

$n = -5/2$, $m = -1/2$, $l = 0$

$$y'' = Ax^{-5/2}y^{-1/2} \tag{16}$$

$n = 1$, $m = -1/2$, $l = 7/5$

$$y'' = Axy^{-1/2}y'^{7/5} \tag{17}$$

$n = -5/2$, $m = -1/2$, $l = 10/7$

$$y'' = Ax^{-5/2}y^{-1/2}y'^{10/7} \tag{18}$$

$(Solutions)_{k1}$ – Table of solution for equations:

For (10)

$$x = \phi(\tau) = aP_3, \qquad y = \psi(\tau) = bP_2^2, \qquad y' = \chi(\tau) = 4/3\tau,$$
$$P_2 = \pm(\tau^2 - 1), \qquad P_3 = \tau^3 - 3\tau + C_2, \qquad A = \pm 4/9a^{-2}b^{3/2};$$

For (12)

$$x = \phi(\tau) = aP_3^{-1}, \qquad y = \psi(\tau) = bP_2^2P_3^{-1}, \qquad y' = \chi(\tau) = \mp 1/3P_4,$$
$$P_4 = \pm(\tau^4 - 6\tau^2 + C_2\tau - 1), \qquad A = \pm 4/9a^{1/2}b^{3/2};$$

For (13)

$$x = \phi(\tau) = aP_2P_3^{-1/2}, \qquad y = \psi(\tau) = bP_4^2, \qquad y' = \chi(\tau) = \pm 16P_3^{5/3},$$
$$A = \pm 15a^{-2}b^{1/2}(a/16b)^{2/5};$$

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