DIFFERENTIAL EQUATIONS
AND
CONTROL PROCESSES
N 2, 2015
Electronic Journal,
reg. N ΦC77-39410 at 15.04.2010
ISSN 1817-2172

http://www.math.spbu.ru/diffjournal
e-mail: jodiff@mail.ru

Stochastic differential equations

ON THE STABILITY OF SOLUTIONS FOR CERTAIN SECOND-ORDER STOCHASTIC DELAY DIFFERENTIAL EQUATIONS

A. M. A. Abou-El-Ela[*1], **A. I. Sadek**[*2] and **A. M. Mahmoud**[**3]
[*]Department of Mathematics, Faculty of Science, Assiut University. Assiut 71516, Egypt.

[**] Department of Mathematics, Faculty of Science, New Valley Branch, Assiut University. New Valley, El-Khargah 72111, Egypt.

Abstract

In this paper, by constructing Lyapunov functionals we investigate sufficient conditions, for the stochastic asymptotic stability of the zero solution of certain second-order stochastic differential equations with delay.

By defining an appropriate Lyapunov functionals, we prove two new theorems on the stochastic asymptotic stability. Our results improve and form a complement to some known recent results in the literature.

***Keywords and phrases*:** Stochastic differential equations, delay, asymptotic stability, Lyapunov functional.

1 Introduction

Modeling of physical systems by ordinary differential equations, ignores stochastic effects. By adding random elements into the differential equations we obtain what is called a stochastic differential equation and the term stochastic is called noise [10].

[1]E-mail address: a.el-ela@aun.edu.eg
[2]E-mail address: Sadeka1961@yahoo.com
[3]Corresponding author. E-mail address: math_ayman27@yahoo.com

Stochastic differential equation is a differential equation, in which one or more of the terms is a stochastic process and resulting in a solution which is itself a stochastic process.

Stochastic differential equations play a relevant role in many application areas including environmental modeling, engineering, biological modeling and mostly, one of more important application of stochastic differential equation is in the modeling of electrical networks.

Due to statistical properties, a stochastic process can be used to define the randomness in an uncorrelated white Gaussian noise, which can be thought of as the derivative of Brownian motion (or the Wiener process) [20].

In many branches of science and industry stochastic delay differential equations have been used to model the evolution phenomena because the measurements of time-involving variables and their dynamics usually contain some delays.

Furthermore stochastic perturbations are often introduced into these deterministic systems in order to describe the effects of fluctuations in real environment, thus yield stochastic delay differential equations.

Mathematically stochastic delay differential equations were first introduced by Itô and Nisio [7], in which the existence and uniqueness of the solutions have been investigated.

In the last several decades, numerous studies have been developed toward the study of stochastic delay differential equations, such as stochastic stability, Lyapunov functional method, Lyapunov exponent, stochastic flow and attraction etc. (see [5], [8], [9], [15], [16], [18], [24] and the references therein).

Stability is one of the most important problems in the study of stochastic delay differential equations. One of the powerful techniques employed in the study of the stability problem is the method of the Lyapunov functionals.

One general method of Lyapunov functionals constructed both for deterministic and stochastic delay differential equations was proposed and developed by many authors, for examples, [12, 13, 19].

The advantage of this method can judge the stability of systems without knowledge of the solutions. Therefore it is at all times hotpot in the study of the stability theory in the last century.

On the other hand, since the introduced stochastic calculus about fifty years ago, the theory of stochastic differential equations have been developed very quickly. In particular, the Lyapunov's second method has been developed to deal with stochastic stability by many authors, for example, [14, 17].

However it is generally much more difficult to construct the Lyapunov functionals in the case of delay, than the Lyapunov functionals in the case of non-delay of higher-order, for example, $[1-3, 22, 23, 25-28]$. Therefore anther useful technique has been developed, that is to compare the stochastic delay differential equations with the corresponding non-delay equations.

Besides it is worth-mentioning that, there are only few papers on the same topic for certain first-order stochastic differential equations with delay, for example, [14, 21].

In [14], Kolmanovskii and Shaikhet present an interesting survey of a general method of Lyapunov functionals construction for stochastic differential equations of neutral type.

In this paper by constructing Lyapunov functionals, we investigate sufficient conditions for the asymptotic stability of the zero solution to the following second-order stochastic delay differential equations

$$\ddot{x}(t) + a\dot{x}(t) + bx(t-h) + \sigma x(t)\dot{\omega}(t) = 0 \tag{1.1}$$

and

$$\ddot{x}(t) + a\dot{x}(t) + f(x(t-h)) + \sigma x(t-\tau)\dot{\omega}(t) = 0, \tag{1.2}$$

where a, b and σ are positive constants, h and τ are two positive constant delays; $\omega(t) \in \mathbb{R}$ is a standard Wiener process; $f(x)$ is a continuous function and $f(0) = 0$.

2 Preliminaries and stability result

A general scalar stochastic differential equation has the following form

$$dx(t) = f(t, x(t))dt + g(t, x(t))d\omega(t), \tag{2.1}$$

where $f(t, x(t)) : [0, T] \times \mathbb{R} \to \mathbb{R}$ and $g(t, x(t)) : [0, T] \times \mathbb{R} \to \mathbb{R}$ are drift and diffusion coefficients and $\omega(t)$ is the so called Wiener process, a stochastic process representing the noise [11].

Instead the appropriate stochastic chain rule, known as Itô formula contains an additional term. This additional term which roughly speaking is due to the fact that the square of the stochastic differential $(d\omega(t))^2$ is equal to dt, in the mean-square sense, i.e. $E[(d\omega(t))^2] = dt$. So the second-order term in $d\omega(t)$ should really appear as a first-order term in dt.

Suppose $x(t)$ is a solution of the stochastic differential equation (2.1) for some suitable functions f, g. Let $g(t, x(t)) : (0, \infty) \times \mathbb{R} \to \mathbb{R}$ be a twice continuously differentiable function.

Let the function

$$y(t) = g(t, x(t)),$$

be a stochastic process, for which

$$dy(t) = \frac{\partial g}{\partial t}(t, x(t))dt + \frac{\partial g}{\partial x}(t, x(t))dx(t) + \frac{1}{2}\frac{\partial^2 g}{\partial x^2}(t, x(t))(dx(t))^2, \qquad (2.2)$$

where $(dx(t))^2 = dx(t).dx(t)$ is computed according to the rules

$$dt.dt = dt.d\omega(t) = d\omega(t).dt = 0, \qquad d\omega(t).d\omega(t) = dt.$$

Hence for any given initial value $x(0) = x_0$ the stochastic differential equation has a unique global solution denoted by $x(t; x_0)$. Assume furthermore that $f(t, 0) = 0$ and $g(t, 0) = 0$ for all $t \geq 0$. Hence the stochastic differential equation admits the zero solution $x(t; 0) \equiv 0$.

Definition 2.1 *The zero solution of the stochastic differential equation is said to be stochastically stable or stable in probability, if for every pair of $\epsilon \in (0, 1)$ and $r > 0$, there exists a $\delta = \delta(\epsilon, r) > 0$ such that*

$$P\{|x(t; x_0)| < r \ \textit{ for all } \ t \geq 0\} \geq 1 - \epsilon, \quad \textit{whenever}\, |x_0| < \delta_0.$$

Otherwise, it is said to be stochastically unstable.

Definition 2.2 *The zero solution of the stochastic differential equation is said to be stochastically asymptotically stable, if it is stochastically stable and moreover for every $\epsilon \in (0, 1)$ and $r > 0$, there exists a $\delta = \delta(\epsilon) > 0$ such that*

$$P\{\lim_{t \to 0} x(t; x_0) = 0\} \geq 1 - \epsilon, \quad \textit{whenever}\, |x_0| < \delta_0.$$

Let K denote the family of all continuous nondecreasing functions $\mu : \mathbb{R}^+ \to \mathbb{R}^+$ such that $\mu(0) = 0$ and $\mu(r) > 0$ if $r > 0$.

For $h > 0$, let $S_h = \{x \in \mathbb{R}^m : |x| < h\}$ and let $C^{1,2}(S_h \times \mathbb{R}^+; \mathbb{R}^+)$ denote the family of all continuous functions $V(t, x)$ from $S_h \times \mathbb{R}^+$ to $\mathbb{R}^+$ with continuous first partial derivatives with respect to every component of x and to t.

We naturally consider the Itô differential of the process $V(t, x(t))$, where $x(t)$ is a solution of the stochastic differential equation and V is a Lyapunov function. According to the Itô formula, we of course require $V \in C^{1,2}(S_h \times \mathbb{R}^+; \mathbb{R}^+)$, which denotes the family of all non-negative functions $V(t, x)$ defined on $S_h \times \mathbb{R}^+$ such that they are twice continuously differentiable in x and once in t.

By the Itô formula we have

$$dV(t, x(t)) = LV(t, x(t))dt + V_x(t, x(t))g(t, x(t))d\omega(t),$$

where

$$LV(t, x(t)) = V_t(t, x(t)) + V_x(t, x(t))f(t, x(t)) + \frac{1}{2}tr[g^T(t, x(t))V_{xx}(t, x(t))g(t, x(t))].$$

We shall see that $\dot{V}(t, x(t))$ will be replaced by the diffusion operator $LV(t, x(t))$ in the study of stochastic stability. For example, the inequality $\dot{V}(t, x(t)) \leq 0$ will sometimes by replaced by $LV(t, x(t)) \leq 0$ to get the stochastic stability.

Theorem 2.1 *Assume that there exist* $V \in C^{1,2}(S_h \times \mathbb{R}^+; \mathbb{R}^+)$ *and* $\mu \in K$ *such that*

$$V(t, 0) = 0, \quad \mu(|x|) < V(t, x) \quad \textit{and}$$
$$LV(t, x(t)) \leq 0, \ \textit{for all} \ (t, x) \in S_h \times \mathbb{R}^+.$$

Then the zero solution of the stochastic differential equation is stochastically stable.

Theorem 2.2 *Assume that there exist* $V \in C^{1,2}(S_h \times \mathbb{R}^+; \mathbb{R}^+)$ *and* $\mu_1, \mu_2, \mu_3 \in K$ *such that*

$$\mu_1(|x|) \leq V(t, x) \leq \mu_2(|x|) \quad \textit{and}$$
$$LV(t, x(t)) \leq -\mu_3(|x|), \ \textit{for all} \ (t, x) \in S_h \times \mathbb{R}^+.$$

Then the zero solution of the stochastic differential equation is stochastically asymptotically stable.

Now we present the main stability results of (1.1) and (1.2).

Theorem 2.3 *Suppose that there exist positive constants* a, b, σ *and* h *with* $ab - \sigma^2 > 0$, *then the zero solution of* (1.1) *is stochastically asymptotically stable, provided that*

$$h < min\left\{\frac{ab - \sigma^2}{ab}, \frac{a}{(4 + a)b}\right\}.$$

Theorem 2.4 *If a, σ, h and τ are positive constants, $\frac{f(x)}{x} \geq b > 0$ for $x \neq 0$, $ab - \sigma^2 > 0$ and there exists $l > 0$ with $|f'(x)| \leq l$, then the zero solution of* (1.2) *is stochastically asymptotically stable, provided that*

$$h < min\left\{\frac{ab - \sigma^2}{al}, \frac{a}{(4+a)l}\right\}.$$

3 Proof of Theorem 2.3.

We write equation (1.1) in the following equivalent form

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -ay - bx + b\int_{t-h}^{t} y(s)ds - \sigma x(t)\dot{\omega}(t). \end{aligned} \tag{3.1}$$

Define the Lyapunov functional $V_1(x_t, y_t)$ as

$$V_1(x_t, y_t) = \frac{1}{2}y^2 + bx^2 + \frac{1}{2}(y + ax)^2 + \gamma\int_{-h}^{0}\int_{t+s}^{t} y^2(u)duds, \tag{3.2}$$

where $x_t = x(t+s),\ s \leq 0$ and γ is a positive constant, which will be determined later.

From (3.2), (3.1) and by using Itô formula, we obtain

$$\begin{aligned} LV_1(x_t, y_t) =& -ay^2(t) - abx^2(t) + 2by\int_{t-h}^{t} y(s)ds + abx\int_{t-h}^{t} y(s)ds + \sigma^2x^2(t) \\ &+ \gamma hy^2(t) - \gamma\int_{t-h}^{t} y^2(u)du, \end{aligned}$$

it follows by using the inequality $2uv \leq u^2 + v^2$,

$$\begin{aligned} LV_1(x_t, y_t) \leq& -ay^2 - abx^2 + bhy^2 + b\int_{t-h}^{t} y^2(s)ds + \frac{abh}{2}x^2 + \frac{ab}{2}\int_{t-h}^{t} y^2(s)ds \\ &+ \sigma^2x^2 + \gamma hy^2 - \gamma\int_{t-h}^{t} y^2(s)ds. \end{aligned}$$

Then we have

$$LV_1(x_t, y_t) \leq -(a - bh - \gamma h)y^2 - (ab - \frac{abh}{2} - \sigma^2)x^2 + (b + \frac{ab}{2} - \gamma)\int_{t-h}^{t} y^2(s)ds.$$

If we take

$$\gamma = \frac{b}{2}(a+2) > 0,$$

then we find

$$LV_1(x_t, y_t) \leq -\{a - bh - \frac{bh}{2}(a+2)\}y^2 - (ab - \frac{abh}{2} - \sigma^2)x^2.$$

Therefore if

$$h < min\left\{\frac{ab - \sigma^2}{ab}, \frac{a}{(4+a)b}\right\},$$

we obtain

$$LV_1(x_t, y_t) \leq -\frac{1}{2}\{ay^2 + (ab - \sigma^2)x^2\}.$$

Then we have

$$LV_1(x_t, y_t) \leq -\alpha(x^2(t) + y^2(t)), \quad \text{for some } \alpha > 0. \tag{3.3}$$

Since $\int_{-h}^{0}\int_{t+s}^{t} y^2(u)duds$ is non-negative, consequently we find

$$V_1(x_t, y_t) \geq \frac{1}{2}y^2 + bx^2 + \frac{1}{2}(y + ax)^2.$$

Then there exists a positive constant D_1 such that

$$V_1(x_t, y_t) \geq D_1(x^2(t) + y^2(t)). \tag{3.4}$$

We can write (3.2) as the following form

$$V_1(x_t, y_t) = y^2 + bx^2 + axy + \frac{1}{2}a^2x^2 + \gamma\int_{-h}^{0}\int_{t+s}^{t} y^2(u)duds.$$

Since $xy \leq \frac{1}{2}(x^2 + y^2)$, then we find

$$V_1(x_t, y_t) \leq y^2 + bx^2 + \frac{a}{2}(x^2 + y^2) + \frac{1}{2}a^2x^2 + \gamma\int_{t-h}^{t}(\theta - t + h)y^2(\theta)d\theta.$$

Therefore we obtain

$$V_1(x_t, y_t) \leq \frac{a+2}{2}\|y\|^2 + \frac{2b + a + a^2}{2}\|x\|^2 + \gamma\frac{h^2}{2}\|y\|^2.$$

Then we have

$$V_1(x_t, y_t) \leq \frac{2b + a + a^2}{2}\|x\|^2 + \frac{a + \gamma h^2 + 2}{2}\|y\|^2.$$

Then we can say that there exists a positive constant D_2, which satisfies

$$V_1(x_t, y_t) \leq D_2(x^2(t) + y^2(t)). \tag{3.5}$$

Hence from (3.3), (3.4) and (3.5) all the conditions of Theorem 2.2 are satisfied, so that the zero solution of (1.1) is stochastically asymptotically stable. Thus the proof of Theorem 2.3 is now complete.

4 Proof of Theorem 2.4.

We write (1.2) as the following equivalent system

$$\begin{aligned} &\dot{x} = y, \\ &\dot{y} = -ay - f(x) + \int_{t-h}^{t} f'(x(s))y(s)ds - \sigma x(t-\tau)\dot{\omega}(t). \end{aligned} \tag{4.1}$$

Define the Lyapunov functional as

$$\begin{aligned} V_2(x_t, y_t) =& 2\int_0^x f(\eta)d\eta + \frac{1}{2}y^2 + \frac{1}{2}(y+ax)^2 + \lambda \int_{-h}^{0}\int_{t+s}^{t} y^2(\theta)d\theta ds \\ &+ \sigma^2 \int_{t-\tau}^{t} x^2(s)ds, \end{aligned} \tag{4.2}$$

where λ is a positive constant, which will be determined later.

From (4.2), (4.1) and by using Itô formula, we have

$$\begin{aligned} LV_2(x_t, y_t) =& -ay^2(t) + 2y\int_{t-h}^{t} f'(x(s))y(s)ds - axf(x) + ax\int_{t-h}^{t} f'(x(s))y(s)ds \\ &+ \sigma^2 x^2(t) + \lambda y^2(t)h - \lambda \int_{t-h}^{t} y^2(\theta)d\theta, \end{aligned}$$

Suppose $|f'(x)| \leq l$, $l > 0$ and $\frac{f(x)}{x} \geq b > 0$, then

$$\begin{aligned} LV_2(x_t, y_t) \leq& -(a-\lambda h)y^2 - abx^2 + \sigma^2 x^2 + 2ly\int_{t-h}^{t} y(s)ds + alx\int_{t-h}^{t} y(s)ds \\ &- \lambda \int_{t-h}^{t} y^2(\theta)d\theta. \end{aligned}$$

Thus by using the inequality $2uv \leq u^2 + v^2$, we get

$$\begin{aligned} LV_2(x_t, y_t) \leq& -(a-\lambda h)y^2 - (ab-\sigma^2)x^2 + lhy^2 + l\int_{t-h}^{t} y^2(s)ds \\ &+ \frac{alh}{2}x^2 + \frac{al}{2}\int_{t-h}^{t} y^2(s)ds - \lambda \int_{t-h}^{t} y^2(\theta)d\theta. \end{aligned}$$

Then we obtain

$$LV_2(x_t, y_t) \leq -(a - lh - \lambda h)y^2 - (ab - \frac{alh}{2} - \sigma^2)x^2 + (l + \frac{al}{2} - \lambda)\int_{t-h}^{t} y^2(s)ds.$$

If we take

$$\lambda = \frac{l}{2}(a+2) > 0,$$

then we get

$$LV_2(x_t, y_t) \le -\{a - 2lh - \frac{lha}{2}\}y^2 - (ab - \frac{alh}{2} - \sigma^2)x^2.$$

Thus if

$$h < min\left\{\frac{ab-\sigma^2}{al}, \frac{a}{(4+a)l}\right\},$$

we find

$$LV_2(x_t, y_t) \le -\frac{1}{2}\{(ab-\sigma^2)x^2 + ay^2\}.$$

Therefore we have

$$LV_2(x_t, y_t) \le -\beta(x^2(t) + y^2(t)), \text{ for some } \beta > 0. \tag{4.3}$$

Since $\int_{-h}^{0}\int_{t+s}^{t} y^2(\theta)d\theta ds$ and $\sigma^2\int_{t-\tau}^{t} x^2(s)ds$ are non-negative, then we find

$$V_2(x_t, y_t) \ge 2\int_0^x f(\eta)d\eta + \frac{1}{2}y^2 + \frac{1}{2}(y+ax)^2.$$

Since $\frac{f(x)}{x} \ge b > 0$, therefore we get

$$V_2(x_t, y_t) \ge bx^2 + \frac{1}{2}y^2 + \frac{1}{2}(y+ax)^2,$$

then there exists a positive constant D_3 such that

$$V_2(x_t, y_t) \ge D_3(x^2(t) + y^2(t)). \tag{4.4}$$

Since $|f'(x)| \le l$ and $f(0) = 0$, then by using the mean-value theorem, we obtain $f(x) \le lx$. Thus we can write (4.2) as the following form

$$V_2(x_t, y_t) \le lx^2 + \frac{1}{2}y^2 + \frac{1}{2}(y+ax)^2 + \lambda\int_{-h}^{0}\int_{t+s}^{t} y^2(\theta)d\theta ds + \sigma^2\int_{t-\tau}^{t} x^2(s)ds.$$

Since $xy \le \frac{1}{2}(x^2+y^2)$, then we find

$$V_2(x_t, y_t) \le lx^2 + y^2 + \frac{a}{2}(x^2+y^2) + \frac{1}{2}a^2x^2 + \lambda\int_{t-h}^{t}(\theta - t + h)y^2(\theta)d\theta$$
$$+ \sigma^2\int_{t-\tau}^{t} x^2(s)ds.$$

Thus we obtain

$$V_2(x_t, y_t) \leq \frac{a+2}{2}\|y\|^2 + \frac{2l+a+a^2}{2}\|x\|^2 + \frac{\lambda h^2}{2}\|y\|^2 + \sigma^2\tau\|x\|^2$$
$$= \frac{2l+a+a^2+2\sigma^2\tau}{2}\|x\|^2 + \frac{a+2+\lambda h^2}{2}\|y\|^2.$$

Hence we have a positive constant D_4 satisfying

$$V_2(x_t, y_t) \leq D_4(x^2(t) + y^2(t)). \tag{4.5}$$

From the results (4.3), (4.4) and (4.5), we note that all the conditions of Theorem 2.2 are satisfied, so that the zero solution of (1.2) is stochastically asymptotically stable.
This completes the proof of Theorem 2.4.

5 Conclusions

If we allow for some randomness in some of the coefficients of a differential equation, we often obtain a more realistic mathematical model of the situation. So the study of stochastic differential equations is richer than the classical deterministic ones.

By constructing Lyapunov functionals, sufficient conditions for the stochastic asymptotic stability of the zero solution of two equations (1.1) and (1.2). The obtained results are extend existing results in the literature on deterministic systems to stochastic delay systems.

Finally, we can say that there exist a lot of applications for the second-order stochastic differential equations with delay or without delay.

The importance of second-order equations is conditioned by Newton second low.

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