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Stohastic differential equations

Existence result for Stochastic Impulsive differential inclusions

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Abstract

We establish the existence of solution of Stochastic impulsive differential inclusion in infinite dimensional space. We employed fixed point theorem for multivalued map to obtain the solution.
AMS Subject Classification: 34K50, 34A60 **Key Words and Phrases:** stochastic differential inclusions, mild solution, impulsive effects.

1 Introduction

Impulsive differential equations model problem with impulsive effects which are due to instantaneous perturbations at certain moments. The vast applications of the theory of impulsive differential equations and inclusions have attracted many authors both to deterministic and stochastic cases. The theory of impulsive differential equations and inclusions were extensively studied in [3] , [8] and the references cited there. Existence of solution for integrodifferential inclusion in infinite dimensional space without impulse effects was established in [4]. This work is concerned with stochastic systems in infinite dimensional space. In [13], concise study of stochastic differential equation in infinite dimensional space was done and this will form a bedrock of this work. By Banach fixed point theorem and semigroup approach, the existence of solution of nonlinear stochastic differential inclusions was established in [1]. Stochastic systems with impulses effects were studied in [12] with non local condition while neutral stochastic evolution inclusions were studied in [11] both for convex and non convex cases.

The existence of weak solution of stochastic differential inclusions has applications in stochastic control and partial differential inclusions as established in [7]and references cited in it. The mild solution of Stochastic evolution inclusions with impulsive effects considered in this work has applications to transition semigroup and hence the Feller property of such semigroup.
The result in this work has applications in the study of control problem for a stochastic dynamical system. The solution sets will be employed in the space of admissible controls for the system and the impulsive effects due to the effects of abrupt interruption of the system over certain period of time. The work also have practical applications in the study of models of interacting species in a random medium in which the impulsive effects are due to some environmental factors over a period of time.
In the sequel, preliminaries necessary for the result shall be stated in section 2 and the main result will be proved in section 3.

2 Preliminaries

Let H be a real separable Hilbert space with inner product $\langle .,. \rangle$ and norm $\| \; . \; \|$ and let K be another real separable Hilbert space with inner product $\langle .,. \rangle_K$ and norm $\| \; . \; \|_K$. $L(K,H)$ denotes the space of bounded operators from K to H. Let $(\Omega, \mathfrak{F}, \mathfrak{F}_t, P)$ be a complete probability space with a filtration $\{\mathfrak{F}_t\}$ satisfying the usual condition (i.e. the filtration contains all P-null sets and is right continuous). $w(t)$ denotes a given K-valued Brownian motion with a finite trace nuclear covariance operator $Q \geq 0$. For $\sigma_1, \sigma_2 \in L(K,H)$, define $\langle\langle \sigma_1, \sigma_2 \rangle\rangle = \text{trace}\langle \sigma_1 Q \sigma_2^* \rangle$, where σ_2^* is the adjoint of the operator σ_2. $L(K,H)$ with the inner product $\langle\langle .,. \rangle\rangle$ is a pre-Hilbert space, its completion with respect to the topology induced by the Q-Hilbert-Schmidt norm $\| \; . \; \|_Q$ $\big(\| \sigma \|_Q^2 = \langle\langle \sigma, \sigma \rangle\rangle < \infty\big)$ is a Hilbert space.
We shall state some definitions and properties of multivalued maps which will be employed in the sequel.
For a nonempty set X, let (X,d) be a metric space and $P(X)$ denote the nonempty family of subsets of X, we shall denote by $P_{cl}(X)$ (resp. $P_{b,cp,cv}(X)$) the nonempty family of closed (resp.bounded, compact and convex)subsets of X.
A multivalued map $F : X \to P(X)$ is closed (resp. convex)-valued if $F(x)$ is closed (resp. convex) for all $x \in X$. F is said to be bounded if for every bounded set $V \in P(X)$, $F(V)$ is bounded in X.

F is said to be upper semicontinuous (u.s.c) at a point point $x \in X$ if $x \in X$, $F(x)$ is a nonempty closed subset of X and for each open set V of X containing $F(x)$ there exists an open neighbourhood N of x such that $F(N) \subset V$. F is said to be u.s.c on a nonempty subset Y of X if F is u.s.c at every point $y \in Y$. If for every bounded subset V of X, $F(V)$ is relatively compact then F is said to be completely continuous. If F is continuous with nonempty compact values, then F is u.s.c if and only if F has closed graph (i.e. $x_n \to x$, $y_n \to y$ implies $y \in F(x)$).

Let $J \subset \mathbb{R}$ be nonempty, a multivalued map $F : J \to P_{bd,cl,cv}(X)$ is said to be measurable if for each $x \in X$ the distance function $Y : J \to \mathbb{R}$ defined by

$$Y(t) = d^*(x, F(t)) = \inf\{d(x, z) : z \in F(t)\}$$

is measurable. All these properties of multivalued maps can be found in [2], [5] and other standard books on set-valued analysis or differential inclusions.

We shall be concerned with the existence of mild solution of the stochastic differential inclusions with impulsive effects:

$$\begin{aligned} & dx(t) \in [Ax(t) + f(t, x(t))]dt + G(t, x(t))dw(t) \quad a.e.\ t \in [0, T],\ t \neq t_k, \quad k = 1, ..., m \\ & \triangle x(t_k) = I_k(x(t_k)), \quad k = 1, ..., m \\ & x(0) = \xi \end{aligned} \tag{2.1}$$

where $\xi \in H$ and A is the infinitesimal generator of an analytic semigroup of bounded linear operators $S(t),\ t \geq 0$,

$$x(t_k^+) = \lim_{h\to 0+} x(t_k + h) \text{ and } x(t_k) = \lim_{h\to 0-} x(t_k + h).$$

The $t_k \geq 0$ are impulsive moments satisfying $t_k < t_{k+1}$ and $\lim_{k\to\infty} t_k = +\infty$. The $\triangle x(t_k) = x(t_k^+) - x(t_k)$ represents the jump in the state at t_k.

We assume that there exists a constant M such that $\| S(t) \|_{B(H)} \leq M$ for $t \in J$ and $0 \in \rho(A)$, where $B(H)$ denotes the space of bounded linear operators on H and $\rho(A)$ is the resolvent set of A.

Definition 2.1 *A multivalued map $F : J \times X \to P(X)$ is said to be L^2-Caratheodory if*

(i) $t \to F(t, u)$ is measurable for each $u \in D$,

(ii) $u \to F(t, u)$ is u.s.c for almost all $t \in J$,

(iii) F is integrably bounded i.e. for each $q > 0$,there exists $h_q \in L^1(J, \mathbb{R}^+)$ such that

$$\| F(t, u) \|^2 = \sup\{\| v \|^2 : v \in F(t, u)\} \leq h(t) \quad for\ all \quad \| u \| \leq q, \quad a.e.\ t \in J.$$

Let $J=[0,b]$ we define $PC \equiv PC\big(J, L^2(\Omega, H)\big)$ as

$$PC = \Big\{\varphi : [0,b] \to H \ such\ that\ \varphi\ is\ a\ continuous\ H-valued\ stochastic\ process\ except\ for\ points\ t_k,\ k=1,...,m;\ at\ which\ \varphi(t_k^-), \varphi(t_k^+)\ exist\ and\ \varphi(t_k^-) = \varphi(t_k)\ a.s.\ \sup_{0\le t\le b} E \mid \varphi(t) \mid^2 < \infty\Big\}$$

PC is a Banach space with norm $\parallel x \parallel_{PC} = \sup_{0\le t\le b}(E \mid x(t) \mid^2)^{\frac{1}{2}}$
For a multivalued map G, we define the set of selections of G; $Q_{G,x}$ as

$$Q_{G,x} = \Big\{v(.) \in L^2(J, L_Q(K,H)) : v(t) \in G(t, x(t))\ a.e\ \ t \in J\Big\}$$

$L^2(J, L_Q(K,H))$ denotes the $L_Q(K,H)$-valued functions $\xi(t)$ satisfying $\int_J \mid \xi(t) \mid_Q^2 dt < \infty$.

Definition 2.2 *By a mild solution of problem (2.1), we mean a process $x \in PC$ such that there exists a selection $v(.) \in Q_{G,x}$ a.e. on J and*

$$x(t) = S(t)\xi + \int_0^t S(t-s)f(s,x(s))ds + \int_0^t S(t-s)v(s)dw(s) + \sum_{0<t_k<t} S(t-t_k)I_k(x(t_k^-)).$$

3 Main Results

The following hypotheses shall be employed in the main result.

Hypothesis 1 (H_1): *$f : J \times H \to H$ is a continuous function; there exists $L_f > 0$ such that*

$$\parallel f(t,x) \parallel \le L_f \parallel x \parallel, \quad x \in H$$

(H_2): *there exists constants $c_k, \bar{c}_k$, $k = 1,...,m$, such that*

$$\parallel I_k(x) - I_k(y) \parallel \le c_k \parallel x - y \parallel,\ for\ any\ x, y \in H\ and\ \parallel I_k(x) \parallel \le \bar{c}_k \parallel x \parallel .$$

(H_3): *$A : D(A) \subset H \to H$ is the infinitesimal generator of a strongly continuous semigroup $S(t)$, $t \ge 0$, which is compact for $t > 0$, and there exists a constant M such that $\parallel S(t) \parallel_{B(H)} \le M$, for each $t \ge 0$.*
(H_4): *the multivalued map $G : J \times H \to P(L_Q(K,H))$ is L_2-Caratheodory, compact and convex. There exists a continuous nondecreasing function $\varphi : [0,\infty) \to (0,\infty)$, $p \in L^1(J, \mathbb{R}^+)$ such that*

$$\parallel G(t,x) \parallel_Q^2 \le p(t)\varphi(\parallel x \parallel^2), \quad for\ a.e.t \in J\ and\ x \in H$$

with

$$\int_0^b m(s)ds < \int_c^\infty \frac{du}{u+\varphi(u)},$$

where

$$m(t) = \max\{M^2L_f^2, M^2p(t)\}, \qquad c = M^2\Big[\parallel \xi \parallel^2 + \sum_{k=1}^m c_k^2\Big]$$

Our main result is based on the following Bohnenblust-Karlin (Corollary 9.8, [15]) Theorem.

Lemma 3.1 *Let X be a Banach space and K be a closed and convex subset of X. Suppose the multivalued map $\Psi : K \to P_{cl,cv}(X)$ is upper semicontinuous and the set $\Psi(K)$ is relatively compact in X. Then Ψ has a fixed point in K.*

We shall employ the following Lasota-Opial result [9].

Lemma 3.2 *Let I be a compact interval and X be a Hilbert space. Let G be a L_2-Caratheodory multivalued map with $Q_{G,x} \neq \emptyset$ and ζ be a linear continuous mapping from $L^2(I,X)$ to $C(I,X)$. Then the operator*

$$\zeta \circ Q_G : C(I,X) \to P_{b,cl,cv}(C(I,X)), \quad x \to (\zeta \circ Q_G)(x) = \zeta(Q_G)(x)$$

is a closed graph operator in $C(I,X) \times C(I,X)$.

We now state and the prove the existence result.

Theorem 3.1 *Assume that conditions H_1, H_2, H_3 and H_4 hold, then problem (2.1) has at least one mild solution.*

Proof 1 *The problem will be transformed to a fixed point problem.*
Define the map $\Phi : PC\big(J, L^2(\Omega, H)\big) \to \mathbb{P}(PC(J, L^2(\Omega, H)))$ by

$$\begin{aligned}\Psi(x) = \Big\{ h \in PC(J, L^2(\Omega, H)) : h(t) = S(t)\xi &+ \int_0^t S(t-s)f(s,x(s))ds \\ &+ \int_0^t S(t-s)v(s)dw(s) \\ &+ \sum_{0<t_k<t} S(t-t_k)I_k(x(t_k^-));\ v(.) \in Q_{G,x} \Big\}\end{aligned} \tag{3.1}$$

Let

$$K = \left\{ x \in PC(J, L^2(\Omega, H)) : \| x \|_{PC}^2 \leq r(t), \quad t \in J \right\}$$

where

$$I(z) = \int_c^z \frac{du}{u + \varphi(u)}, \qquad I(r(t)) = \int_0^t m(s) ds$$

K is a closed and bounded convex subset of $PC(J, L^2(\Omega, H))$. Let $k^* = \sup\{\| x \|_{PC}^2 : x \in K\}$. We have to show that $\Psi(K) \subset K$, Ψ is relatively compact and upper semicontinuous. The proof is divided to steps.

***Step 1**: $\Psi(K) \subset K$.*

For a fixed $t \in J$, let $x \in K$ be arbitrarily chosen, we show that $\Psi(x) \in K$. For each $t \in K$, there exists $v(.) \in Q_{G,x}$

$$h(t) = S(t)\xi + \int_0^t S(t-s) f(s, x(s)) ds + \int_0^t S(t-s) v(s) dw(s) + \sum_{0<t_k<t} S(t-t_k) I_k(x(t_k^-));$$

Then

$$\begin{aligned}
\| h(t) \|^2 &= \sup_{t \in J} E \mid h(t) \mid^2 \\
&\leq \sup_{t \in J} \Bigg(E \mid S(t)\xi \mid^2 + \int_0^t E \mid S(t-s) f(s, x(s)) \mid^2 ds \\
&+ \int_0^t E \mid S(t-s) v(s) \mid^2 dw(s) + \sum_{0<t_k<t} E \mid S(t-t_k) I_k(x(t_k^-)) \mid^2 \Bigg) \\
&\leq M^2 \| \xi \|^2 + M^2 \sum_{k=1}^m c_k^2 + \int_0^t M^2 L_f^2 \| x(s) \|^2 + M^2 p(s) \varphi(\| x(s) \|^2) ds \\
&\leq M^2 \| \xi \|^2 + M^2 \sum_{k=1}^m c_k^2 + \int_0^t m(s)(\| x(s) \|^2 + \varphi(\| x(s) \|^2)) ds \\
&\leq c + \int_0^t m(s)(r(s) + \varphi(r(s))) ds \\
&= c + \int_0^t r'(s) ds \\
&= r(t), \qquad \left(since \quad \int_c^{r(s)} \frac{du}{u + \varphi(u)} = \int_0^s m(\tau) d\tau \right)
\end{aligned}$$

*Hence $\Psi(x) \in K$. Since it is true for any $x \in K$, $\Psi(K) \subset K$. Therefore $\Psi : K \to K$. **Step 2.**: $\Psi(K)$ is relatively compact.*

Let $\tau_1, \tau_2 \in J$, $\tau_1 < \tau_2$, *and* $\epsilon > 0$ *with* $0 < \epsilon \le \tau_1 < \tau_2$. *Let* $x \in K$ *and* $h \in \Psi(x)$. *Then there exists* $v \in Q_{G,x}$ *such that for each* $t \in J$ *we have*

$$\begin{aligned}
\| \ h(\tau_2) - h(\tau_1) \ \|^2 &= E \sup_{t\in J} \mid h(\tau_2) - h(\tau_1) \mid^2 \\
&\le \sup_{t\in J} \Bigg(E \mid S(\tau_2)\xi - S(\tau_1)\xi \mid^2 \\
&+ \int_0^{\tau_1-\epsilon} E \mid (S(\tau_2 - s) - S(\tau_1 - s)) f(s, x(s)) \mid^2 ds \\
&+ \int_{\tau_1}^{\tau_1-\epsilon} E \mid (S(\tau_2 - s) - S(\tau_1 - s)) f(s, x(s)) \mid^2 ds \\
&+ \int_{\tau_1}^{\tau_2} E \mid S(\tau_2 - s) f(s, x(s)) \mid^2 ds \\
&+ \int_0^{\tau_1-\epsilon} E \mid (S(\tau_2 - s) - S(\tau_1 - s)) v(s) \mid^2 dw(s) \\
&+ \int_{\tau_1}^{\tau_1-\epsilon} E \mid (S(\tau_2 - s) - S(\tau_1 - s)) v(s) \mid^2 dw(s) \\
&+ \int_{\tau_1}^{\tau_2} E \mid S(\tau_2 - s) v(s) \mid^2 dw(s) \\
&+ M^2 c_k^2 (\tau_2 - \tau_1) + \sum_{0<t_k<\tau_1} \big(E \mid (S(\tau_2 - t_k) - S(\tau_1 - t_k)) I_k(x_{t_k^-}) \mid^2 \big) \Bigg) \\
&\le \| \ S(\tau_2)\xi - S(\tau_1)\xi \ \|^2 \\
&+ \int_0^{\tau_1-\epsilon} \| \ (S(\tau_2 - s) - S(\tau_1 - s)) f(s, x(s)) \ \|^2 ds \\
&+ \int_{\tau_1}^{\tau_1-\epsilon} \| \ (S(\tau_2 - s) - S(\tau_1 - s)) f(s, x(s)) \ \|^2 ds \\
&+ \int_{\tau_1}^{\tau_2} \| \ S(\tau_2 - s) f(s, x(s)) \ \|^2 ds \\
&+ \int_0^{\tau_1-\epsilon} \| \ (S(\tau_2 - s) - S(\tau_1 - s)) \ \|^2 \| \ v(s) \ \|^2 ds \\
&+ \int_{\tau_1}^{\tau_1-\epsilon} \| \ (S(\tau_2 - s) - S(\tau_1 - s)) \ \|^2 \| \ v(s) \ \|^2 ds \\
&+ \int_{\tau_1}^{\tau_2} \| \ S(\tau_2 - s) \ \|^2 \| \ v(s) \ \|^2 ds \\
&+ M^2 c_k^2 (\tau_2 - \tau_1) + \sum_{0<t_k<\tau_1} c_k^2 \ \| \ S(\tau_2 - t_k) - S(\tau_1 - t_k) \ \|^2
\end{aligned}$$

The right hand side tends to zero as $\tau_2 \to \tau_1$ and for ϵ sufficiently small, since $S(t)$ is a strongly continuous operator and compactness of $S(t)$, for $t > 0$, implies the continuity in the uniform operator topology. This proves the equicontinuity for the case $t \neq t_i$, $i = 1, ..., m$. To prove the equicontinuity at $t = t_i$, by Arzela-Ascoli theorem, it suffices to show that Ψ maps K into a precompact set in $L^2(\Omega, H)$. Let $0 < t \leq b$ be fixed and let ϵ be a real number satisfying $0 < \epsilon < t$. For $x \in K$, we define

$$\begin{aligned} h_\epsilon(t) = S(t)\xi + S(\epsilon) \int_0^{t-\epsilon} S(t-s-\epsilon) f(s, x(s)) ds \\ + S(\epsilon) \int_0^{t-\epsilon} S(t-s-\epsilon) v(s) ds \\ + S(\epsilon) \sum_{0<t_k<t-\epsilon} S(t-t_k-\epsilon) I_k(x(t_k^-)) \end{aligned}$$

where $v \in Q_{G,x}$. Since $S(t)$ is a compact operator, the set $H_\epsilon(t) = \{h_\epsilon(t) : h_\epsilon \in \Psi(x)\}$ is precompact in $L^2(\Omega, H)$ for every ϵ, $0 < \epsilon < t$. Moreover, for every $h \in \Psi(x)$, we have

$$\begin{aligned} \parallel h_\epsilon(t) - h(t) \parallel^2 \leq L_f^2 k^* \int_{t-\epsilon}^t \parallel S(t-s) \parallel^2 ds \\ + \int_{t-\epsilon}^t \parallel S(t-s) \parallel^2 \mid r(s) \mid ds \\ + \sum_{t-\epsilon \leq t_k < t} c_k^2 \parallel S(t-t_k) \parallel^2 \end{aligned}$$

Therefore there are precompact sets arbitrarily close to the set $\{h(t) : h \in \Psi(x)\}$. Hence the set $\{h(t) : h \in \Psi(x)\}$ is precompact in $L^2(\Omega, H)$.
***Step 3**: To show that Ψ is upper semicontinuous, it suffices to show that it has a closed graph.*
Let $x_n \to x_*$, $h_n \in \Psi(x_n)$ and $h_n \to h_*$. We will prove that $h_* \in \Psi(x_*)$.
$h_n \in \Psi(x_n)$ means that there exists $v_n \in Q_{G(x_n)}$ such that, for each $t \in J$,

$$\begin{aligned} h_n(t) = S(t)\xi + \int_0^t S(t-s) f(s, x_n(s)) ds + \int_0^t S(t-s) v_n(s) ds \\ + \sum_{0<t_k<t} S(t-t_k) I_k(x_n(t_k^-)). \end{aligned}$$

We must prove that there exists $v_* \in Q_{G(x_*)}$ *such that, for each* $t \in J$

$$h_*(t) = S(t)\xi + \int_0^t S(t-s)f(s,x_*(s))ds + \int_0^t S(t-s)v_*(s)ds + \sum_{0<t_k<t} S(t-t_k)I_k(x_*(t_k^-)).$$

Consider the linear and continuous operator $\rho : L^2(J, L_Q(K,H)) \to PC(J, L^2(\Omega, H))$, *defined by*

$$(\rho v)(t) = \int_0^t S(t-s)v(s)dw(s).$$

We have

$$\begin{aligned} E \mid \big(h_n(t) - S(t)\xi - \int_0^t S(t-s)f(s,x_n(s))ds \\ - \sum_{0<t_k<t} S(t-t_k)I_k(x_n(t_k^-))\big) - \big(h_*(t) - S(t)\xi \\ - \int_0^t S(t-s)f(s,x_*(s))ds - \sum_{0<t_k<t} S(t-t_k)I_k(x_*(t_k^-))\big) \mid^2 \\ = E \mid h_n(t) - h_*(t) + \int_0^t S(t-s)\big(f(s,x_n(s)) - f(s,x_*(s))\big)ds \mid^2 \\ \leq 2E \mid h_n - h_* \mid^2 + 2L_f^2 \int_0^t \parallel x_n - x_* \parallel^2 \to 0 \quad as\ n \to \infty. \end{aligned}$$

Now, we have

$$h_n(t) - S(t)\xi - \int_0^t S(t-s)f(s,x_n(s))ds \in \rho \circ Q_{G(x_n)}$$

from Lemma 3.2, so $\rho \circ Q_G$ *is a closed graph operator.*
Since $x_n \to x_*,\ h_n \to h_*$, *there exist* $v_* \in Q_{G,x_*}$ *such that*

$$h_* - S(t)\xi - \int_0^t S(t-s)f(s,x_*(s))ds = \int_0^t S(t-s)v_*(s)dw(s)$$

Then Ψ *is upper semicontinuous and by Lemma 3.1, there exists a fixed point which is a mild solution of problem (2.1)*

Example

Consider for problem (2.1), $G(t,X)$ which is lower semicontinuous and has continuous selection $g(X(t))$ and let $f(t,X) \equiv f(X(t))$. We have

$$\begin{aligned} dX(t) &= [AX(t) + f(X(t))]dt + g(X(t))dw(t) \quad a.e.\ t \in [0,T],\ t \neq t_k, \quad k = 1,...,m \\ \triangle X(t_k) &= I_k(X(t_k)), \quad k = 1,...,m \\ X(0) &= \xi \in H \end{aligned} \tag{3.2}$$

The mild solution of the problem (3.2) is a process $X(t)$ such that

$$X(t) = S(t)\xi + \int_0^t S(t-s)f(X(s))ds + \int_0^t S(t-s)g(s)dw(s) + \sum_{0<t_k<t} S(t-t_k)I_k(X(t_k^-)).$$

We define the transition semigroup corresponding to this mild solution as follows:

Let $B_b(H) = \{\psi : H \to \mathbb{R}; \psi$ is bounded and Borel measurable$\}$. For any $\varphi \in B_b$, we define

$$P_t\varphi(\xi) = \mathbb{E}(\varphi(X(\xi)(t))), \quad t \in [0,T], \quad \xi \in H, \quad t \neq t_k$$

Then P_t, $t \in [0,T]$, $t \neq t_k$ has a semigroup property, i.e. for all $\xi \in H$, and $s,t \in [0,T]$, $s,t \neq t_k$ with $s+t \in [0,T]$, $(s+t) \neq t_k$.

Under appropriate assumptions, the function $P_t\varphi$ is globally Lipschitz in H, which means that the semigroup is strong Feller [6] .

References

[1] Ahmed, N. U. Non linear stochastic differential inclusions on Banach space, *Stoch. Anal. Appl.* 12, (1994),1-10.

[2] Aubin , J. P. and Cellina, A. *Differential Inclusions*, Springer- Verlag, Berlin, 1984.

[3] Benchohra, M. , Henderson, J. and Ntouyas, S.K. *Impulsive differential equations and inclusions* Hindawi Publishing Corporation, vol 2., New York, 2006.

[4] Cernea, A. On an Evolution inclusion in non-separable Banach spaces *Opuscula Mathematica*, 2, 131-138, (2009).

[5] Deimling, K.*Multivalued differential equations*, Walter de Gruyter, 1992.

[6] Es-Sarhir, A. Existence and uniqueness of invariant measures for a class of transition semigroups on Hilbert spaces *J. Math. Anal Appl.*, 353, 497-507, (2009).

[7] Kisielewicz, M. Stochastic representation of partial differential inclusions *J. Math. Anal Appl.*, 353, (2009), 592-606.

[8] V. Lakshmikantham, V., Bainov, D. D. and Simenov, P. S. *Theorem of Impulsive Differential Equations* , World Scientific, Singapore, 1989.

[9] A. Lasota, A. and Opial, Z. An application of the Kakutani-Ky-Fan theorem in the theory of ordinary differential equations. *Bull. Acad. Pol. Sci. Ser. Sci. Math. Astronom. Phys.,* 13, (1965), 781-786.

[10] Michael, E. Continuous selections, I and II, *Annals of Math.* 63; 64, (1956), 361-382, 562-580.

[11] Ning, H. W. and Liu, B. Existence results for impulsive neutral stochastic evolution inclusions in Hilbert space, *Acta Math. Sinica, English series* 27, 7, (2011), 1405-1418.

[12] Pan, L. Existence of mild solution for impulsive stochastic differential equations with nonlocal conditions *Diff. Equat. and Appl.* 4, 3, (2012), 485-494.

[13] Prato, G. D. and Zabczyk, J. *Stochastic Equations in Infinite Dimensions*, Cambridge University Press, Cambridge, 1992.

[14] Shen, L. J. and Sun, J. T. Existence and uniqueness of mild solutions for non linear stochastic impulsive differential equation, *Abstract and Applied Anal.* vol. 2011, ID 439724,(2011), 1-10.

[15] E. Zeidler, E. *Nonlinear Functional Analysis and Its Applications, I. Fixed-Point Theorems*, Springer, New York, 1986.