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Finding Determinant for Some Block Band Matrices by Reblocking

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Abstract

In this note, we present a reliable process for computing the determinant of any general block pentadiagonal and block heptadiagonal matrices with (or without) corners. Then we extend this process for finding determinant of block anti-pentadiagonal and block anti-heptadiagonal matrices. We do this work by determinant of a block tridiagonal matrix and expansions $det(X) = exp(trace(log(X)))$.

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Keywords: Determinant, Tridiagonal, Pentadiagonal, Heptadiagonal, Block Matrix.

1 Introduction

Consider the equation $M\Psi = 0$,

$$M = \begin{pmatrix} A_1 & B_1 & D_1 & & & E_{-1} & C_0 \\ C_1 & A_2 & B_2 & D_2 & & & E_0 \\ E_1 & C_2 & A_3 & B_3 & D_3 & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & E_{n-4} & C_{n-3} & A_{n-2} & B_{n-2} & D_{n-2} \\ D_{n-1} & & & E_{n-3} & C_{n-2} & A_{n-1} & B_{n-1} \\ B_n & D_n & & & E_{n-2} & C_{n-1} & A_n \end{pmatrix} \tag{1}$$

or

$$M = \begin{pmatrix} A_1 & B_1 & D_1 & E_1 & & & & & S_{-2} & Q_{-1} & C_0 \\ C_1 & A_2 & B_2 & D_2 & E_2 & & & & & S_{-1} & Q_0 \\ Q_1 & C_2 & A_3 & B_3 & D_3 & E_3 & & & & & S_0 \\ S_1 & Q_2 & C_3 & A_4 & B_4 & D_4 & E_4 & & & & \\ & S_2 & Q_3 & C_4 & A_5 & B_5 & D_5 & E_5 & & & \\ & & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & & \\ & & & S_{n-6} & Q_{n-5} & C_{n-4} & A_{n-3} & B_{n-3} & D_{n-3} & E_{n-3} \\ E_{n-2} & & & & S_{n-5} & Q_{n-4} & C_{n-3} & A_{n-2} & B_{n-2} & D_{n-2} \\ D_{n-1} & E_{n-1} & & & & S_{n-4} & Q_{n-3} & C_{n-2} & A_{n-1} & B_{n-1} \\ B_n & D_n & E_n & & & & S_{n-3} & Q_{n-2} & C_{n-1} & A_n \end{pmatrix} \tag{2}$$

where elements of matrix M in (1) and (2) are matrices $m \times m$ and $\Psi \in \mathbb{C}^{nm}$ is unknown. These equations applied for the thigh binding model for a crystal, a molecule, or a particle in a lattice with random potential or hopping amplitudes.

In [5, 7], Molinari and Salkuyeh proposed different processes for finding determinant of BTD with(or without) corners. In this note, we present approximations for the determinant of block pentadiagonal and block heptadiagonal matrices with(or without) corners by reblocking and expansions of $det(X) = exp(trace(log(X)))$.

We expand this process for block anti-pentadiagonal and anti-heptadiagonal matrix with(or without) corners.

We do this work by determinant of a block tridiagonal matrix. This work has the ability to save time and memory specially for some large block band matrices. This process can use for parallel computing and solving differential equations using finite differences.

The rest of this paper is organized as follows: in the next section, we present a process for finding determinant of general block pentadiagonal and block heptadiagonal matrices. In section 3 we expand this method for block band matrices with corners. Finally compute an approximation for determinant by $det(X) = exp(trace(log(X)))$.

1.1 Notation

We will often (but not always)use the following convention:

BTD for block tridiagonal matrix.

BPD for block pentadiagonal matrix.
BHD for block heptadiagonal matrix.
BAPD for block anti-pentadiagonal matrix.
BAHD for block anti-heptadiagonal matrix.
log(X) and exp(X) denote logarithm and exponential function of a matrix.
ln(x) and e^x denote the natural logarithm and exponential function of a scalar x. The eigenvalues of a complex square matrix T are $\lambda_j(T)$ and its spectral radius is $\rho(T) \equiv max_j \mid \lambda_j(T) \mid$.
I_n (or simply I if its dimension is clear) is the $n \times n$ identity matrix.

2 Computing the Determinant for Block Pentadiagonal and Block Heptadiagonal Matrices

In [5, 7], are proposed different processes for finding determinant of BTD. We try to change these process for BPD and BHD. Also we extend this process for BAPD and BAHDs.
Consider determinant for matrix M,

$$M = \begin{pmatrix} A_1 & B_1 & D_1 & & & \\ C_1 & A_2 & B_2 & D_2 & & \\ E_1 & C_2 & A_3 & B_3 & D_3 & \\ & \ddots & \ddots & \ddots & \ddots & \ddots \\ & & E_{n-3} & C_{n-2} & A_{n-1} & B_{n-1} \\ & & & E_{n-2} & C_{n-1} & A_n \end{pmatrix} \tag{3}$$

or

$$M = \begin{pmatrix} A_1 & B_1 & D_1 & E_1 & & & & & & \\ C_1 & A_2 & B_2 & D_2 & E_2 & & & & & \\ Q_1 & C_2 & A_3 & B_3 & D_3 & E_3 & & & & \\ S_1 & Q_2 & C_3 & A_4 & B_4 & D_4 & E_4 & & & \\ & S_2 & Q_3 & C_4 & A_5 & B_5 & D_5 & E_5 & & \\ & & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & & S_{n-6} & Q_{n-5} & C_{n-4} & A_{n-3} & B_{n-3} & D_{n-3} & E_{n-3} \\ & & & & S_{n-5} & Q_{n-4} & C_{n-3} & A_{n-2} & B_{n-2} & D_{n-2} \\ & & & & & S_{n-4} & Q_{n-3} & C_{n-2} & A_{n-1} & B_{n-1} \\ & & & & & & S_{n-3} & Q_{n-2} & C_{n-1} & A_n \end{pmatrix} \tag{4}$$

Suppose N be a nonsingular matrix similar:

$$N = \begin{pmatrix} G_1 & H_1 & & & & \\ F_1 & G_2 & H_2 & & & \\ & F_2 & G_3 & H_3 & & \\ & & \ddots & \ddots & \ddots & \\ & & & F_{l-2} & G_{l-1} & H_{l-1} \\ & & & & F_{l-1} & G_l \end{pmatrix} \tag{5}$$

Salkuyeh [7] by the sequence of matrices:

$$N_i = \begin{pmatrix} G_1 & H_1 & & & & \\ F_1 & G_2 & H_2 & & & \\ & F_2 & G_3 & H_3 & & \\ & & \ddots & \ddots & \ddots & \\ & & & F_{i-2} & G_{i-1} & H_{i-1} \\ & & & & F_{i-1} & G_i \end{pmatrix}, \qquad i = 2, \ldots, l \tag{6}$$

proposed a different procedure for the evaluation of the determinant of N_i:
Suppose $N_1 = G_1$,
$N_i = \begin{pmatrix} N_{i-1} & \hat{H}_{i-1} \\ \hat{F}_{i-1} & G_i \end{pmatrix}, \qquad i = 2, \ldots, l$
where
$\hat{F}_{i-1} = (\underbrace{0 \;\; 0 \;\; \ldots \;\; 0}_{i-2} \quad F_{i-1})^T$
$\hat{H}_{i-1} = (\underbrace{0 \;\; 0 \;\; \ldots \;\; 0}_{i-2} \quad H_{i-1}^T)^T$
By the block LU factorization [1] of N_i can be written:

$$N_i = \begin{pmatrix} N_{i-1} & \hat{H}_{i-1} \\ \hat{F}_{i-1} & G_i \end{pmatrix} = \begin{pmatrix} I & 0 \\ \hat{F}_{i-1}N_{i-1}^{-1} & I \end{pmatrix} \begin{pmatrix} N_{i-1} & \hat{H}_{i-1} \\ 0 & G_i - \hat{F}_{i-1}N_{i-1}^{-1}\hat{H}_{i-1} \end{pmatrix}$$

Hence

$$det(N_i) = det(N_{i-1})det(G_i - \hat{F}_{i-1}N_{i-1}^{-1}\hat{H}_{i-1}) \tag{7}$$

For computing N_{i-1}^{-1} use of BSI Algorithm in [6].
Molinari [5] described a transfer matrix T that built as the product of l matrices of size $(2m \times 2m)$ for finding det(N), [G_i, H_{i-1} *and* F_{i-1} $(i = 1, \ldots, l)$ *are complex and nonsingular* $m \times m$ *matrices*]. Now by transfer

matrix in [5], where $T(i)$ is the partial product of i matrices, we have:

$$T(i)=\begin{pmatrix}-H_i^{-1}G_i & -H_i^{-1}F_{i-1}\\ I_{2m} & 0\end{pmatrix}T(i-1),\quad T(0)_{11}=I_{2m}\ ,\ T(1)_{11}=-H_1^{-1}G_1$$

$$T(i)_{11}=-H_i^{-1}G_iT(i-1)_{11}-H_i^{-1}F_{i-1}T(i-2)_{11}$$
$$det(N)=det[T_{11}(l)]det(H_1\dots H_{l-1}) \tag{8}$$

Theorem 1 *For a BPD similar matrix M in Equation(3), when A_i, D_i and E_i are nonsingular blocks, we can find determinant matrix M by a BTD.*

Proof. *We can make some partitions similar:*

$$F_i=\begin{pmatrix}E_{2i-1} & C_{2i}\\ 0 & E_{2i}\end{pmatrix},G_i=\begin{pmatrix}A_{2i-1} & B_{2i-1}\\ C_{2i-1} & A_{2i}\end{pmatrix},H_i=\begin{pmatrix}D_{2i-1} & 0\\ B_{2i} & D_{2i}\end{pmatrix},$$

$$i=1,2\dots,l\ \ that\ \ l=\frac{n}{2}$$

This work helps us for finding $det(M)$ by BTD N in Equations(5), (7) and (8):

$$det(M)=det(N)$$

Theorem 2 *For a BHD similar matrix M in Equation(4), when A_i , E_i and S_i are nonsingular blocks, we can find determinant of matrix M by a BTD.*
Proof. *For matrix M and ($i=1,2\dots,l\ \ that\ \ l=\frac{n}{3}$), suppose:*

$$F_i=\begin{pmatrix}S_{3i-2} & Q_{3i-1} & C_{3i}\\ 0 & S_{3i-1} & Q_{3i}\\ 0 & 0 & S_{3i}\end{pmatrix},G_i=\begin{pmatrix}A_{3i-2} & B_{3i-2} & D_{3i-2}\\ C_{3i-2} & A_{3i-1} & B_{3i-1}\\ Q_{3i-2} & C_{3i-1} & A_{3i}\end{pmatrix},H_i=\begin{pmatrix}E_{3i-2} & 0 & 0\\ D_{3i-1} & E_{3i-1} & 0\\ B_{3i} & D_{3i} & E_{3i}\end{pmatrix}$$

Now by BTD N in Equations(5), (7) and (8), we have:

$$det(M)=det(N)$$

Remark. If n is even for Theorem 1 or n mod 3 is zero for Theorem 2, we can use above partitions.

In many of applications such as parallel computing and differential equations using finite differences, determinant and inverse of general anti-pentadiagonal(AP) or anti-heptadiagonal(AH) matrices are required.

$$AP = \begin{pmatrix} & & & & D_1 & B_1 & A_1 \\ & & & D_2 & B_2 & A_2 & C_1 \\ & & D_3 & B_3 & A_3 & C_2 & E_1 \\ & \vdots & \vdots & \vdots & \vdots & & \\ B_{n-1} & A_{n-1} & C_{n-2} & E_{n-3} & & & \\ A_n & C_{n-1} & E_{n-2} & & & & \end{pmatrix} \quad (9)$$

$$AH = \begin{pmatrix} & & & & & & E_1 & D_1 & B_1 & A_1 \\ & & & & & E_2 & D_2 & B_2 & A_2 & C_1 \\ & & & & E_3 & D_3 & B_3 & A_3 & C_2 & Q_1 \\ & & & E_4 & D_4 & B_4 & A_4 & C_3 & Q_2 & S_1 \\ & & & \vdots & \vdots & \vdots & & & & \\ B_{n-1} & A_{n-1} & C_{n-2} & Q_{n-3} & S_{n-4} & & & & & \\ A_n & C_{n-1} & Q_{n-2} & S_{n-3} & & & & & & \end{pmatrix} \quad (10)$$

For finding determinant of matrices AP and AH, we use the permutation matrices.

For permutation matrix Q, we can proof:

$$Q = Inverse(Q) = Transpose(Q)$$

see [2]. Also for finding determinant of BAPD and BAHD, we use of permutation matrix:

$$P = AP \times Q_{n-block \times n-block} \quad and \quad H = AH \times Q_{n-block \times n-block}$$

Transfer BAPD and BAHD to BTD, by block permutation matrices and devised matrices in Theorems 1 and 2.

3 Computing the Determinant for some Block band Matrices with corners

Theorem 3 *For a BPD with corners similar matrix M in Equation (1), when A_i , E_i and D_i, $(i = 0, 1, 2 \ldots, l$ that $l = \frac{n}{2})$ are nonsingular, we can find* *determinant of matrix M by a BTD, when n is even.*

Proof. *We can make some partitions similar:*

$$F_i = \begin{pmatrix} E_{2i-1} & C_{2i} \\ 0 & E_{2i} \end{pmatrix}, G_i = \begin{pmatrix} A_{2i-1} & B_{2i-1} \\ C_{2i-1} & A_{2i} \end{pmatrix}, H_i = \begin{pmatrix} D_{2i-1} & 0 \\ B_{2i} & D_{2i} \end{pmatrix}$$

$$i = 0, 1, 2 \ldots, l \ \ that \ \ l = \frac{n}{2}$$

Reblocking helps us for finding $det(M)$ *by the following BTD:*

$$N = \begin{pmatrix} G_1 & H_1 & & & & F_0 \\ F_1 & G_2 & H_2 & & & \\ & F_2 & G_3 & H_3 & & \\ & & \ddots & \ddots & \ddots & \\ & & & F_{l-2} & G_{l-1} & H_{l-1} \\ H_l & & & & F_{l-1} & G_l \end{pmatrix} \tag{11}$$

Theorem 4 *For a BHD similar matrix M in Equation(2), when* A_i , E_i *and* S_i, $(i = 0, 1, 2 \ldots, l \ \ that \ \ l = \frac{n}{3})$, *are nonsingular, we can find determinant of matrix M by a BTD. For matrix M suppose:*

$$F_i = \begin{pmatrix} S_{3i-2} & Q_{3i-1} & C_{3i} \\ 0 & S_{3i-1} & Q_{3i} \\ 0 & 0 & S_{3i} \end{pmatrix}, G_i = \begin{pmatrix} A_{3i-2} & B_{3i-2} & D_{3i-2} \\ C_{3i-2} & A_{3i-1} & B_{3i-1} \\ Q_{3i-2} & C_{3i-1} & A_{3i} \end{pmatrix}, H_i = \begin{pmatrix} E_{3i-2} & 0 & 0 \\ D_{3i-1} & E_{3i-1} & 0 \\ B_{3i} & D_{3i} & E_{3i} \end{pmatrix}$$

$$(i = 0, 1, 2 \ldots, l \ \ that \ \ l = \frac{n}{3})$$

Now by BTD in Equation(11), we have: $det(M) = det(N)$

By above theorems we can find determinant for Matrix M in Equations(1),(2) by BTD. We consider the determinant for block tridiagonal matrix N in Equation(11) by method in [5]. In this method transfer matrix T built as the product of l matrices of size $(2m \times 2m)$ for finding det(N).

$$T = \begin{pmatrix} -H_l^{-1} G_l & -H_l^{-1} F_{l-1} \\ I_m & 0 \end{pmatrix} \cdots \begin{pmatrix} -H_1^{-1} G_1 & -H_1^{-1} F_0 \\ I_m & 0 \end{pmatrix} \tag{12}$$

The transfer matrix is nonsingular, since $detT = \cap_{i=1}^{l} det(H_i^{-1} F_{i-1})$ then we can proof:

$$detN = (-1)^{lm-m} exp(\sum_{i=1}^{2m} ln(1 - \lambda_i(T))) det(H_1 \ldots H_l) \tag{13}$$

Theorem 5 *Let T be a complex matrix of order n, with $\|T\| < 1$ and $(\lambda_i, i = 1, 2 \ldots, n)$ are eigenvalues of T then*

$$det(I - T) = exp(\sum_{i=1}^{n} ln(1 - \lambda_i))$$

Proof. If $\|T\| < 1$ then $I - T$ is nonsingular and $\rho(T) < 1$ [1].
We have $det(I - T) = exp(trace(log(I - T)))$ see [3].
From the linearity of the trace [4] and the fact that $trace(T^p) = \sum_{i=1}^{n} \lambda_i(T)^p$, also by power series for any λ_i with $|\lambda_i| < 1$ follow:

$$log(I - T) = log(I + (-T)) = \sum_{p=1}^{\infty} \frac{(-1)^{p-1}}{p}(-T)^p = -\sum_{p=1}^{\infty} \frac{T^p}{p}$$

$$trace(log(I - T)) = -\sum_{p=1}^{\infty} \frac{1}{p} trace(T^p) = \sum_{i=1}^{n} \sum_{p=1}^{\infty} \frac{-1}{p} \lambda_i(T)^p = \sum_{i=1}^{n} ln(1 - \lambda_i) \tag{14}$$

so

$$det(I - T) = exp(trace(log(I - T))) = exp(\sum_{i=1}^{n} ln(1 - \lambda_i)) \tag{15}$$

Corollary 1 *If matrix N in Equation(11) is nonsingular and $\rho(T) < 1$, we have*

$$detN = (-1)^{lm-m} exp(\sum_{i=1}^{2m} ln(1 - \lambda_i)) det(H_1 \ldots H_l) \tag{16}$$

Suppose every block in matrix N is $m \times m$.
Proof. If N is nonsingular so $T - I$ is nonsingular (Lemma 1 in [5]). Since matrix T is $2m \times 2m$ then $det(I - T) = det(T - I)$ and $det(I - T) = exp(trace(log(I - T)))$. Now by $\rho(T) < 1$ and proof of Theorem(5), we have Equation(16).

Example 1 We try to find determinant of matrix M (pentadiagonal matrix with corners) by reblocking and above theorem.

$$M = \begin{pmatrix} -0.0100 & 0.0200 & 0.2200 & 0 & 0.0100 & -0.0200 \\ 0.0030 & 0.0040 & 0.0700 & 0.0090 & 0.0030 & -0.0040 \\ -0.1000 & -0.0200 & 0.5500 & 0.0600 & 0.5000 & 0 \\ 0 & -0.0400 & 0.0700 & 0.0800 & 0.0200 & 0.0600 \\ 0.0600 & -0.5000 & 0.2000 & -0.0030 & 0.0500 & 0.1000 \\ 0.2000 & 0.1000 & 0 & -0.0070 & 0.0110 & 0.0120 \end{pmatrix}$$

or

$$M = \begin{pmatrix} A_1 & B_1 & C_0 \\ C_1 & A_2 & B_2 \\ B_3 & C_2 & A_3 \end{pmatrix}$$

that

$$A_1 = \begin{pmatrix} -.01 & .02 \\ .003 & .004 \end{pmatrix}, A_2 = \begin{pmatrix} .55 & .06 \\ .07 & .08 \end{pmatrix}, A_3 = \begin{pmatrix} .05 & .10 \\ .011 & .012 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} .22 & 0 \\ .07 & .009 \end{pmatrix}, B_2 = \begin{pmatrix} .5 & 0 \\ .02 & .06 \end{pmatrix}, B_3 = \begin{pmatrix} .06 & -.5 \\ .2 & .1 \end{pmatrix}$$

$$C_0 = \begin{pmatrix} .01 & -.02 \\ .003 & -.004 \end{pmatrix}, C_1 = \begin{pmatrix} -.1 & -.02 \\ 0 & -.04 \end{pmatrix}, C_2 = \begin{pmatrix} .2 & -.003 \\ 0 & -.007 \end{pmatrix}$$

We compute matrix T by Equation(12).

$$T = \begin{pmatrix} -B_3^{-1}A_3 & -B_3^{-1}C_2 \\ I & 0 \end{pmatrix} \begin{pmatrix} -B_2^{-1}A_2 & -B_2^{-1}C_1 \\ I & 0 \end{pmatrix} \begin{pmatrix} -B_1^{-1}A_1 & -B_1^{-1}C_0 \\ I & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -0.1748 & -0.0425 & 0.0030 & -0.0601 \\ 0.1816 & 0.0451 & -0.0111 & 0.0773 \\ 0.2324 & 0.1085 & 0.0476 & -0.0685 \\ 0.7853 & 0.3864 & 0.0102 & 0.2669 \end{pmatrix}$$

Then use of Equation (16) in Corollary 1 so:

$$detM = (-1)^6 exp(\sum_{i=1}^{4} ln(1 - \lambda_i(T)))det(B_1 B_2 B_3) = 5.0730e - 006$$

If set $H_l := -H_l$ and $F_0 := -F_0$ in matrix N of Equation(11) and $-1 < \lambda_i(T) < 1, 1 \leq i \leq 2m$, then

$$detN = (-1)^{lm} det(I+T) det(H_1 \ldots H_l) \tag{17}$$

that

$$det(I+T) = exp(\sum_{i=1}^{2m} ln(1+\lambda_i(T))) \tag{18}$$

see [3, 5].

Example 2 If set $C_0 := -C_0$ and $B_3 := -B_3$ in Matrix M of Example 1, by eigenvalues of transfer matrix T: $(0.2242, -0.0690, 0.0148+0.0305i, 0.0148-0.0305i)$ and by Equation(17) , we have

$$det(M) = (-1)^6 exp(\sum_{i=1}^{4} ln(1+\lambda_i(T))) det(B_1 B_2 B_3) = 7.3971e-006$$

For block anti-pentadiagonal and block anti-heptadiagonal matrices with corners similar:

$$AP = \begin{pmatrix} E_{-1} & C_0 & & & & & D_1 & B_1 & A_1 \\ E_0 & & & & D_2 & B_2 & A_2 & C_1 & \\ & & D_3 & B_3 & A_3 & C_2 & E_1 & & \\ & \vdots & \vdots & \vdots & \vdots & & & & \\ B_{n-1} & A_{n-1} & C_{n-2} & E_{n-3} & & & & & D_{n-1} \\ A_n & C_{n-1} & E_{n-2} & & & & & B_n & D_n \end{pmatrix} \tag{19}$$

and

$$AH = \begin{pmatrix} S_{-2} & Q_{-1} & C_0 & & & & E_1 & D_1 & B_1 & A_1 \\ S_{-1} & Q_0 & & & & E_2 & D_2 & B_2 & A_2 & C_1 \\ S_0 & & & & E_3 & D_3 & B_3 & A_3 & C_2 & Q_1 \\ & & & E_4 & D_4 & B_4 & A_4 & C_3 & Q_2 & S_1 \\ & & & & \vdots & \vdots & \vdots & & & \\ B_{n-1} & A_{n-1} & C_{n-2} & Q_{n-3} & S_{n-4} & & & & D_{n-1} & E_{n-1} \\ A_n & C_{n-1} & Q_{n-2} & S_{n-3} & & & & B_n & D_n & E_n \end{pmatrix} \tag{20}$$

use of permutation matrix similar matrix Q, ($P = AP \times Q_{n-block \times n-block}$ *and* $H = AH \times Q_{n-block \times n-block}$), see [2].

Namely transfer BAPD and BAHD to BPD and BHD, by block permutation matrix Q then find determinant of them by devised matrices of Theorems 3 and 4.

Example 3 For finding determinant of the following matrix:

$$M_1 = \begin{pmatrix} -0.0200 & 0.0100 & 0 & 0.2200 & 0.0200 & -0.0100 \\ -0.0040 & 0.0030 & 0.0090 & 0.0700 & 0.0040 & 0.0030 \\ 0 & 0.5000 & 0.0600 & 0.5500 & -0.0200 & -0.1000 \\ 0.0600 & 0.0200 & 0.0800 & 0.0700 & -0.0400 & 0 \\ 0.1000 & 0.0500 & -0.0030 & 0.2000 & -0.5000 & 0.0600 \\ 0.0120 & 0.0110 & -0.0070 & 0 & 0.1000 & 0.2000 \end{pmatrix}$$

We use of permutation matrix Q and some results of this paper:

$$Q = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

and

$$M_2 = M_1 \times Q = \begin{pmatrix} -0.0100 & 0.0200 & 0.2200 & 0 & 0.0100 & -0.0200 \\ 0.0030 & 0.0040 & 0.0700 & 0.0090 & 0.0030 & -0.0040 \\ -0.1000 & -0.0200 & 0.5500 & 0.0600 & 0.5000 & 0 \\ 0 & -0.0400 & 0.0700 & 0.0800 & 0.0200 & 0.0600 \\ 0.0600 & -0.5000 & 0.2000 & -0.0030 & 0.0500 & 0.1000 \\ 0.2000 & 0.1000 & 0 & -0.0070 & 0.0110 & 0.0120 \end{pmatrix}$$

We gain determinant of matrix M_2 by applied process in Example 1 and determinant of matrix Q, so

$$detM_1 = -(-1)^6 exp(\sum_{i=1}^{4} ln(1-\lambda_i(T)))det(B_1B_2B_3) = -5.0730e - 006$$

4 Summary

In this paper, we present a sequence of approximations for determinant of BPD, BHD, BAPD and BAHD with(or without) corners, this work is done by BTD and expansions $det(X) = exp(trace(log(X)))$.
Introduced processes in this paper try to find determinant for some block band matrices by determinant of a transfer matrix with smaller rank. This work can save time and memory specially for some large band matrices. We can apply this process for parallel computing, telecommunication system analysis and in solving differential equations using finite differences.

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